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FISCAL POLICY IN A MONETARIST MODEL

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ABSTRACT

In a model which exhibits many monetarist properties it is shown that monetary and fiscal policies must be coordinated. The model is populated by overlapping generations of three-period lived agents who can hold fiat money, fiat bonds, and physical capital. A government produces a public good and issues fiat money and fiat bonds to finance permanent budget deficits. In this model both fiat money and fiat bonds can have value in equilibrium, and their co-existence can allow a more efficient financing of deficits than can a single debt instrument.

The views expressed herein are solely those of the author and do not necessarily represent the views of the Senate Budget Committee, the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

FISCAL POLICY IN A MONETARIST MODEL

I. Introduction and Summary of Results

In a model which exhibits many monetarist properties it is shown that monetary and fiscal policies must be coordinated. If the monetary authorities were to be adhere to a path of moderate money growth, the government would have to adopt a correspondingly tight budget policy or it would face insolvency. This finding suggests that a joint policy of slowing money growth to combat inflation and reducing tax revenues to spur economic growth simply is not feasible.

The model developed in this paper is based on a theory of individual optimizing behavior in a dynamic, certain setting. In this model both fiat money and fiat bonds can have value in equilibrium, and their coexistence can allow a more efficient tax structure. The model extends the work of Marco Martins (1980) based on considerations raised in Bryant and Wallace (1980).

In Martin's model individuals of a given generation live for three periods and the generations overlap. In the first period of life individuals are endowed with a nonstorable good which can be consumed or exchanged for money and bonds. While money can be exchanged for goods in any period, bonds must be held for two periods before they have value. Thus, if both money and bonds are held in equilibrium, bonds must bear a positive nominal interest rate. This rate corresponds to the value of "waiting."

Because there is no storable good or capital in Martin's model, bonds need compete only with money. It follows that the nominal interest rate is determined solely by the ratio of bonds to money and is independent of the inflation rate.

Martin's theory of the interest rate depends crucially on two assumptions. First, it is implicitly assumed that a type of arbitrage is pro-

hibited. To illustrate, suppose the current young could buy from the current middle-aged bonds which mature in one period. Then both bonds and money would be held in equilibrium only if the nominal rate of return on bonds were zero. As long as the two-period rate of return on bonds were positive, such transactions could permit each party to earn a certain and positive one-period rate of return on bonds, in which case bonds would dominate money.

The second crucial assumption is that no storage or investment of goods is possible. The nominal interest rate is independent of the inflation rate in Martin's model, because bonds are not substitutes for capital in investors' portfolios.

The present analysis extends Martin's work by relaxing these two assumptions. Capital is introduced into the model with a storage technology that matches the payout pattern of bonds: zero gross return after one period and a positive gross return after two periods. The model then is examined under a portfolio autarky regime in which neither bonds nor claims on capital can be traded and under a laissez-faire regime in which all trades are allowed.

It is shown that the portfolio autarky regime dominates the laissez-faire regime. A broader class of budget deficit policies can be financed under the portfolio autarky regime, and any deficit which can be financed under laissez-faire can be financed more efficiently under portfolio autarky. The restrictions on trade increase efficiency because they allow the government to discriminate in its taxes between individuals' second- and third-period consumption. Thus, the portfolio autarky regime cannot be dismissed based on grounds of Pareto inefficiency.^{1/}

^{1/}The dominance of portfolio autarky over laissez-faire was found also in Bryant-Wallace (1980).

Under portfolio autarky many monetarist propositions follow. First, the rate of inflation is equal to the growth rate of money. Second, an exact Fisher effect obtains, so that the nominal interest rate on bonds is equal to the real rate of return on capital plus the rate of inflation. Third, there is a crowding out effect so that it sometimes is possible to finance increased government spending by bond issue without raising inflation. Finally, when the real rate of return on bonds is nonnegative, an open market purchase of securities by the monetary authority leads to a higher price level.

Two interesting policy implications emerge under the portfolio autarky regime. The first is that monetary and fiscal policies must be coordinated. If the government adopts a larger real deficit policy, the monetary authority cannot adhere to the same time path of money. As bond issue is increased in order to finance a larger deficit, the net interest on the debt increases, so that the same common rate of growth for money and bonds cannot allow the budget with added interest payments to be financed.

The second interesting implication is that a policy which increases government borrowing when the real interest rate is positive benefits the current old and middle-aged generations, but makes the current young and future generations worse off. This standard result obtains even though the government does not raise explicit taxes in the future to pay off the interest on the debt. The sale of bonds initially allows a lower price level, since bonds have a lower transactions velocity than money. Thus, the old and middle-aged can buy more goods with their existing holdings of money and bonds. With a positive real rate of return on bonds, however, bond issue is a negative source of revenue for the government. In order to finance the initial real deficit and the added net real interest payments on the debt, greater use of the inflation tax is required. This then reduces the amount of goods that

the current young and all future generations can purchase in any future period with given real money balances.

The physical environment, which is independent of monetary regime, is described in Section 2. In the next section the portfolio autarky model is developed, and its solution is discussed. In the fourth section properties of feasible policies under portfolio autarky are described, and the effects of alternative policies on macro variables and welfare are examined. The analysis is conducted under different assumptions about the value of the real rate of return on capital. In the fifth section the laissez-faire model is described, and the dominance of portfolio autarky over laissez-faire is demonstrated. In the concluding section arguments are advanced for why the main conclusions can be expected to hold under more general formulations. Assuming they do hold, this work then has implications for some current policy issues. First, it implies that a monetarist prescription to reduce the growth rate of money is a reasonable anti-inflation policy only when it forces a corresponding tightening in fiscal policy. Second, it implies that the continued easing of regulations in the financial industry may not be desirable.

II. The Physical Environment

Time is discrete and runs from $t=2, \dots$. In each period t , $N(t)$ individuals are born. These individuals live for three periods, so that the population in period t consists of the old $N(t-2)$, the middle-aged $N(t-1)$, and the young $N(t)$. The set of individuals in generation t is denoted $\{N(t)\}$, and it is assumed $N(t) = N$ for all t .

A. Individual Preferences

Each individual $h \in \{N(t)\}$ has preferences over his or her lifetime consumption of private goods and public goods $\langle C_1^h(t), C_2^h(t), C_3^h(t) \rangle$ and $\langle G(t), G(t+1), G(t+2) \rangle$, respectively, which can be represented by a utility function separable in C and G and log-linear with respect to C :

$$U\{C_1^h(t), C_2^h(t), C_3^h(t); G(t), G(t+1), G(t+2)\} = \\ \ln C_1^h(t) + \beta \ln C_2^h(t) + \gamma \ln C_3^h(t) + V(G(t), G(t+1), G(t+2)),$$

where $C_i^h(t)$ is the real consumption of the private good by individual $h \in \{N(t)\}$ in the i^{th} period of life, $i=1, 2, 3$, and $G(t)$ is the real amount of the public good produced in period t .

While the particular form of the utility function is assumed for mathematical convenience, it also leads to some definite qualitative results where a more general utility function could cause them to be ambiguous. Thus, the implications of the model should be interpreted as possible results rather than as logical deductions assuming nicely behaved preferences.

B. Monetary and Fiscal Policies

The government produces a constant amount of the public good over time, $G(t) = G$, employing a one-for-one transformation process with inputs being the private goods taxed away from the private sector. The value of G is taken as a parameter and represents the real government deficit net of real interest payments. It could be considered the amount of real spending in excess of that which can be financed through explicit taxation. Thus, G can be financed only through money or debt issue.

In order to inquire about the relationship between monetary and fiscal policies, it is necessary to distinguish between the total units of bonds issued by the government in period t , $\tilde{B}(t)$, and the total units of bonds purchased by the private sector in period t , $B(t)$. One unit of bonds issued in period t promises to pay the holder nothing in period $t+1$ and to pay one dollar in period $t+2$. The bonds issued in period t sell at a price in terms of dollars of $v(t)$. Fiscal policy consists of the level of the real government deficit G and the amount of bonds sold each period on the open mar-

ket, $\{G, \langle \tilde{B}(t) \rangle_{t=2,3,\dots}\}$. The government must issue enough bonds each period so that its real expenditures are balanced by its real revenues:

$$(1) \quad G + p(t)B(t-2) = p(t)v(t)\tilde{B}(t), \quad t=2, 3, \dots,$$

where $p(t)$ is the amount of commodities which can be purchased with one dollar (the inverse of the price level), real expenditures include the real spending on public goods G and the retirement of privately held bonds $p(t)B(t-2)$, and real revenues consist of the resources raised through sales of bonds on the open market $p(t)v(t)\tilde{B}(t)$.

Monetary policy is conducted by a separate authority which buys bonds on the open market by printing money $M(t)$. Thus, monetary policy can be identified with the sequence $\langle M(t) \rangle$, $t=2, \dots$, and the sequence must satisfy

$$(2) \quad v(t)[\tilde{B}(t)-B(t)] = M(t) - M(t-1), \quad t=2, 3, \dots,$$

where $v(t)[\tilde{B}(t)-B(t)]$ is the dollar value of bonds purchased by the monetary authority and $M(t) - M(t-1)$ is the amount of money printed. It is being assumed that the monetary authority does not demand payment from the government for its maturing bonds: it just accumulates bonds in its portfolio.

A consolidated government-monetary authority budget constraint can be derived by solving for $\tilde{B}(t)$ in (2) and substituting the resultant expression into (1):

$$(3) \quad G + p(t)B(t-2) = p(t)[v(t)B(t)+M(t)-M(t-1)], \quad t=2, 3, \dots,$$

where the left-hand side is the amount of real government expenditures and the right-hand side is the amount of real resources the government raises from the private sector through issuing bonds and printing money.

The consolidated budget constraint is essentially the one found in Martins and Bryant-Wallace.

C. Returns on Investments

Individuals can purchase money or bonds or store capital. Let $I > 0$ be a given amount of goods to be invested at time t . The return streams on capital, bonds, and money are as follows:

<u>If I is invest- ed at time t in</u>	<u>at time t+1 it is worth</u>	<u>at time t+2 it is worth</u>
Capital: I	0	$X(t)I$
Bonds: $I = p(t)v(t)b^h(t)$	0	$p(t+2)b^h(t)$
Money: $I = p(t)m^h(t)$	$p(t+1)m^h(t)$	$p(t+2)m^h(t)$

The one- and two-period gross rates of return on capital, bonds, and money then are:

<u>Asset</u>	<u>1 period gross rate of return</u>	<u>2 period gross rate of return</u>
Capital	0	$X(t)$
Bonds	0	$\frac{p(t+2)}{p(t)v(t)}$
Money	$\frac{p(t+1)}{p(t)}$	$\frac{p(t+2)}{p(t)}$

The two-period gross rates of return on money and bonds are defined by:

$$R_1^2(t) \equiv \frac{p(t+2)}{p(t)} \text{ and } R_2(t) \equiv \frac{p(t+2)}{p(t)v(t)}.$$

Only solutions to the models which imply constant rate of return over time will be considered, so the rates can be written $X(t) = X$, $R_1(t) = R_1$, and $R_2(t) = R_2$.

D. Individual Endowments

The current young and all future generations are endowed in the first period of life with y units of consumption goods and nothing in the second and third periods:

$$\langle W_1^h(t), W_2^h(t), W_3^h(t) \rangle = \langle y, 0, 0 \rangle, \quad t=2, 3, \dots, y > 0$$

where $W_1^h(t)$ is the endowment in terms of goods of individual $h \in \{N(t)\}$ in his or her i^{th} period of life.

The current old, $h \in \{N(0)\}$, enter period 2, their last period of life, with an endowment of maturing bonds and private goods which are shared equally among members of the generation:

$$W_3^h(0) = p(2)b^h(0) + Xk^h(0),$$

where

$$b^h(0) = B(0)/N,$$

$$k^h(0) = K(0)/N, \text{ and}$$

$B(0) > 0$, $K(0) > 0$ are given.

The current middle-aged, $h \in \{N(1)\}$, enter period 2, their second period of life, with an endowment of money shared equally:

$$W_2^h(1) = p(2)m^h(1),$$

where

$$m^h(1) = M(1)/N \text{ and}$$

$M(1) > 0$ is given.

An individual $h \in \{N(1)\}$ enters period 3 with an endowment of maturing bonds and private goods which are shared equally with other individuals of that generation:

$$w_3^h(1) = p(3)b^h(1) + xk^h(1),$$

where

$$b^h(1) = B(1)/N,$$

$$k^h(1) = K(1)/N,$$

$K(1) > 0$ is given, and

$B(1) > 0$ is determined by current policy.

III. Equilibrium Under Portfolio Autarky

Under portfolio autarky no private exchanges of bonds or capital are allowed. Individuals of one generation cannot borrow from individuals of another generation using the returns on bonds or capital to repay the debt. Stated another way, individuals cannot sell claims in a given period to bonds or capital maturing in the next period.

Given their endowments and faced with sequences of prices $p(t)$ and $v(t)$, individuals maximize the utility of private consumption $\ln C_1^h(t) + \beta \ln C_2^h(t) + \gamma \ln C_3^h(t)$ by choosing how much to save out of their endowments and how to invest the savings among money, bonds, and capital.

The maximization problems of the current old, the current middle-aged, and the current young and all future generations can be written:

$$(4) \quad h \in \{N(0)\}: \text{maximize } \ln C_3^h(0) \text{ with respect to } C_3^h(0)$$

subject to

$$C_3^h(0) \leq W_3^h(0) = p(2)b^h(0) + Xk^h(0)$$

$$(5) \quad h \in \{N(1)\}: \text{maximize } \beta \ln C_2^h(1) + \gamma \ln C_3^h(1) \text{ with respect to } C_2^h(1), C_3^h(1)$$

subject to

$$a. \quad C_2^h(1) \leq W_2^h(1) = p(2)m^h(1)$$

$$b. \quad C_3^h(1) \leq W_3^h(1) = p(3)b^h(1) + Xk^h(1)$$

$$(6) \quad h \in \{N(t)\}, t \geq 2: \text{maximize } \ln C_1^h(t) + \beta \ln C_2^h(t) + \gamma \ln C_3^h(t)$$

$$\text{with respect to } C_1^h(t), C_2^h(t), C_3^h(t)$$

subject to

$$a. \quad C_1^h(t) \leq y - p(t)m^h(t) - p(t)v(t)b^h(t) - k^h(t)$$

$$b. \quad C_2^h(t) \leq p(t+1)m^h(t)$$

$$c. \quad C_3^h(t) \leq p(t+2)b^h(t) + Xk^h(t).$$

Three notes are in order. First, since the log-linear utility function implies nonsatiation, all budget constraints can be taken as equalities. Second, the current old and the current middle-aged have the trivial decisions to consume all that their endowments allow. Thus, their individual demand functions for consumption are:

$$\hat{C}_3(0) = p(2)B(0)/N + XK(0)/N,$$

$$\hat{C}_2(1) = p(2)M(1)/N, \text{ and}$$

$$\hat{C}_3(1) = p(3)B(1)/N + XK(1)/N.$$

Third, for the current young and all future generations, it is being assumed that at least one of the two-period rates of return on bonds and capital, R_2 and X , respectively, exceeds the two-period rate of return on money, R_1^2 . Otherwise, money--and not bonds or capital--would be held for third-period consumption.

Solution of problem (6) yields the steady-state individual demand functions for consumption, money, bonds, and capital. An immediate implication of problem (6) is that bonds will be demanded only if $R_2 > X$; otherwise capital dominates. Similarly, bonds and capital will both be demanded only if $R_2 = X$. Define the two-period rates of inflation, nominal interest, and net return on physical capital, Π , r , and ρ , respectively, by

$$R_1^2 = \frac{p(t+2)}{p(t)} = \frac{1}{1+\Pi},$$

$$R_2 = \frac{p(t+2)}{p(t)v(t)} = \frac{1+r}{1+\Pi}, \text{ and}$$

$$X = 1 + \rho.$$

Constant R_1 and R_2 , then, translate into constant rates of inflation and nominal interest. The assumption that bonds are held for third-period consumption implies $R_2 > R_1^2$ or $r > 0$. Both bonds and capital are held only if $R_2 = X$ or $\frac{1+r}{1+\Pi} = 1 + \rho$. This latter condition is a discrete time version of an exact Fisher effect and can be written $r = \rho + \Pi + \rho\Pi \approx \rho + \Pi$, that is, the

nominal rate of interest on bonds is (except for a cross-product term) the sum of the real rate of return on capital and the rate of inflation.

The budget equations (6)a, b, and c can be collapsed into a single equation. Write equation (6)a as:

$$(6)a' \quad C_1 = y - I_1 - I_2,$$

where the dependence of C_1 on h and t has been suppressed,

$$I_1 = p(t)m^h(t) \text{ and}$$

$$I_2 = p(t)v(t)b^h(t) + k^h(t).$$

I_1 and I_2 represent the real amounts invested for second- and third-period consumption, respectively. Equation (6)b can be written:

$$C_2 = \left(\frac{p(t+1)}{p(t)}\right)p(t)m^h(t) = R_1 I_1,$$

so that

$$I_1 = \frac{C_2}{R_1}.$$

Equation (6)c can be written:

$$C_3 = \left(\frac{p(t+2)}{p(t)v(t)}\right)p(t)v(t)b^h(t) + Xk^h(t) = R_2 p(t)v(t)b^h(t) + Xk^h(t).$$

If both $b^h(t)$ and $k^h(t)$ are positive, $R_2 = X$ and $C_3 = R_2 I_2$. If $R_2 > X$, then

$$k^h(t) = 0, \quad I_2 = p(t)v(t)b^h(t),$$

and again

$$C_3 = R_2 I_2.$$

Thus,

$$I_2 = \frac{C_3}{R_2}.$$

Substituting for I_1 and I_2 in (6)a', the representative individual's maximization problem can be expressed

maximize $\ln C_1 + \beta \ln C_2 + \gamma \ln C_3$ with respect to C_1, C_2, C_3

subject to

$$C_1 + C_2/R_1 + C_3/R_2 = y.$$

The budget constraint is simply that the present value of consumption is equal to the present value of income.

Solution of this problem yields the following individual steady-state demand functions for consumption:

$$\hat{C}_1 = \frac{y}{1+\beta+\gamma},$$

$$\hat{C}_2 = \frac{\beta R_1 y}{1+\beta+\gamma}, \text{ and}$$

$$\hat{C}_3 = \frac{\gamma R_2 y}{1+\beta+\gamma}.$$

Let $\hat{m}^d(R_1, R_2)$, $\hat{b}^d(R_1, R_2)$, and $\hat{k}^d(R_1, R_2)$ be the steady-state demands at time t of an individual in generation t for money, bonds, and capital, respectively. All demand functions are in terms of time t goods. Since $C_2 = R_1 p(t) m^h(t)$, we have:

$$\hat{m}^d(R_1, R_2) \equiv p(t) m^h(t) = \frac{\hat{C}_2}{R_1} = \frac{\beta y}{1+\beta+\gamma}.$$

If $R_2 > X$, we have:

$$C_3 = R_2 p(t) v(t) b^h(t),$$

so that

$$\hat{b}^d(R_1, R_2) \equiv p(t) v(t) \hat{b}^h(t) = \frac{\hat{C}_3}{R_2} = \frac{\gamma y}{1 + \beta + \gamma} \text{ and}$$

$$\hat{k}^d(R_1, R_2) \equiv \hat{k}^h(t) = 0.$$

Finally, if $R_2 = X$, the individual is indifferent between holding bonds and holding capital:

$$\hat{b}^d(R_1, R_2) + \hat{k}^d(R_1, R_2) \equiv p(t) v(t) \hat{b}^h(t) + \hat{k}^h(t) = \frac{\hat{C}_3}{R_2} = \frac{\gamma y}{1 + \beta + \gamma}.$$

Let $\alpha \in [0, 1]$ be the share of third-period consumption financed by bonds, so that $\alpha = 1$ if $R_2 > X$ and α is arbitrary if $R_2 = X$. Then,

$$\hat{b}^d(R_1, R_2) = \frac{\alpha \gamma y}{1 + \beta + \gamma} \text{ and}$$

$$\hat{k}^d(R_1, R_2) = \frac{(1 - \alpha) \gamma y}{1 + \beta + \gamma}.$$

Since individuals in each generation are identical, aggregate demand functions are just N times individual demand functions.

A stationary equilibrium consists of sequences of prices $p(2)^*$, $p(3)^*$, ..., and $v(2)^*$, $v(3)^*$, ..., such that:

- a. the rate of inflation Π and the nominal rate of interest r are constant over time,

b. the aggregate demands for money, bonds, and goods equal the respective aggregate supplies:

i. $\hat{N}_m^h(t) = M(t), t=2, \dots$

ii. $\hat{N}_b^h(t) = B(t), t=2, \dots$

iii. $\hat{N}_C_3^h(t-2) + \hat{N}_C_2^h(t-1) + \hat{N}_C_1^h(t) + \hat{N}_k^h(t) +$

$$G(t) = N_y + XK(t-2), t=2, 3, \dots, \text{ and}$$

c. each individual maximizes utility subject to given endowments and prices $p(2)^*, v(2)^*, \dots$

It is convenient to substitute the government budget constraint for

biii.^{2/} At $t = 2$, we have

$$G + p(2)B(0) = p(2)[vB(2)+M(2)-M(1)].$$

At $t = 3$, we have:

$$G + p(3)B(1) = p(3)[vB(3)+M(3)-M(2)].$$

For $t \geq 4$, we have:

$$G + p(t)B(t-2) = vp(t)B(t) + p(t)M(t) - p(t)M(t-1) \text{ or}$$

$$G + \left[\frac{p(t)}{p(t-2)v} \right] p(t-2)vB(t-2) =$$

$$B^d(R_1, R_2) + M^d(R_1, R_2) - \frac{p(t)}{p(t-1)} p(t-1)M(t-1)$$

^{2/}It is straightforward to show that biii and the government budget constraint are equivalent, given bi, bii, and satisfaction of individual budget constraints.

where

$$p(t)M(t) = N_p(t)\hat{m}^h(t) \equiv M^d(R_1, R_2),$$

$$vp(t)B(t) = N_{vp}(t)\hat{b}^h(t) \equiv B^d(R_1, R_2), \text{ and}$$

the equalities follow from bi and bii.

The steady-state budget constraint for $t > 4$, thus, can be written:

$$G = (1-R_1)M^d(R_1, R_2) + (1-R_2)B^d(R_1, R_2),$$

which states that the real government deficit G is financed by the tax on real money holdings $(1-R_1)M^d(R_1, R_2)$ and the tax on real bond holdings $(1-R_2)B^d(R_1, R_2)$. The tax on money holdings raises revenues only if $R_1 < 1$, the inflation rate is positive, and the tax on bond holdings raises revenue only if $R_2 < 1$, the real net rate of return on bonds is negative.

We are now in position to characterize equilibrium under portfolio autarky. We first characterize it when no capital is held and then modify the characterization to allow for capital holdings. Define

$$S_A(G) = \{(R_1, R_2) \mid (1-R_1)M^d(R_1, R_2) + (1-R_2)B^d(R_1, R_2) = G,$$

$$R_2 > X \text{ and } R_2 > R_1^2 > 0, \text{ and}$$

$$(M^d(R_1, R_2), B^d(R_1, R_2)) > (0, 0)\}.$$

Proposition 1

If

- (a) $(R_1, R_2) \in S_A(G)$ and

(b) $B(0) + M(1) > 0$, then an equilibrium is given by the $\{p(t)\}$, $\{v(t)\}$, $\{M(t)\}$, and $\{B(t)\}$ solutions to

- (i) $\frac{p(t+1)}{p(t)} = R_1, t \geq 2$
- (ii) $v(t) = R_1^2/R_2, t \geq 2$
- (iii) $p(t)M(t) = M^d(R_1, R_2), t \geq 2$
- (iv) $v(t)p(t)B(t) = B^d(R_1, R_2), t \geq 2$

with initial conditions

$$(v) \quad p(2) = \frac{B^d(R_1, R_2) + M^d(R_1, R_2) - G}{B(0) + M(1)}$$

$$(vi) \quad B(1) = \frac{[B(0)+M(1)][B^d(R_1, R_2)+(1-R_1)M^d(R_1, R_2)-G]}{R_1[B^d(R_1, R_2)+M^d(R_1, R_2)-G]}$$

The proof is straightforward:

The conditions (i) - (vi) and the assumption that (R_1, R_2) is in $S_A(G)$ ensure that the resulting $p(t)$ and $v(t)$ sequences clear the money and bond markets and satisfy the government budget constraint for each $t \geq 2$. For a given R_1 and R_2 ,

- (v) determines the initial price $p(2)$,
- (i) determines prices at every future date $t=3, 4, \dots$,
- (ii) determines v , a constant for all time,
- (iii) determines the path of money for all time $t=2, 3, \dots$, and
- (iv) determines the path of privately held bonds for all time, $t=2, 3, \dots$

A few notes are in order. First, when $R_2 > X$, the real aggregate demand function for bonds $B^d(R_1, R_2)$ is well defined and is simply \hat{NC}_3/R_2 . In

this case, the demand for capital is zero. Second, the total supply of government bonds $\tilde{B}(t)$ can be computed using the constraint for open market operations (2):

$$\tilde{B}(t) = B(t) + \frac{1}{v}[M(t)-M(t-1)], \quad t=2, 3, \dots$$

Third, the conditions of Proposition 1 imply that a stationary R_1 and R_2 policy is equivalent to choosing an initial ratio of money to bonds and a constant growth rate over time of money and bonds:

$$\frac{p(2)M(2)}{vp(2)B(2)} = \frac{M^d(R_1, R_2)}{B^d(R_1, R_2)},$$

which can be written

$$\frac{M(2)}{B(2)} = \frac{R_1^2}{R_2} \frac{M^d(R_1, R_2)}{B^d(R_1, R_2)},$$

and in our case reduces to

$$\frac{M(2)}{B(2)} = \left(\frac{1}{1+r}\right) \frac{\beta}{\gamma},$$

so that the ratio of privately held money to privately held bonds determines the nominal interest rate; and

$$\frac{p(t+1)M(t+1)}{p(t)M(t)} = \frac{vp(t+1)B(t+1)}{vp(t)B(t)} = 1, \quad t=2, 3, \dots,$$

or

$$\frac{M(t+1)}{M(t)} = \frac{B(t+1)}{B(t)} = \frac{p(t)}{p(t+1)} = \frac{1}{R_1} = (1+\pi)^{1/2},$$

so that the growth rate of money and the growth rate of bonds are equal to each other and to the rate of inflation. Finally, $B(1)$, the endowment of bonds to the current middle-aged is policy dependent. It is defined so that $\frac{p(3)}{p(2)} = R_1$. If it were not defined this way, changing the ratio of money to bonds and the constant growth rate of money and bonds would not lead to a stationary change in R_1 and R_2 . If we assume $R_1(t)$ and $R_2(t)$ are independent of time, we have

$$p(3) = \frac{B^d(R_1, R_2) + M^d(R_1, R_2) - G}{B(1) + M(2)}.$$

But now suppose there is a change in policies which causes a change in R_1 and R_2 . If $B(1)$ were held constant, we no longer can have $\frac{p(3)}{p(2)} = R_1$, so that the current young would not be looking at the new R_1 and R_2 and, thus, R_1 and R_2 would have to vary over time. Alternative policies, then, must be thought of as initial ratios of money to bonds, constant growth rates over time of money and bonds, and an initial transfer of bonds to the current middle-aged.

Equilibrium under portfolio autarky with $R_2 = X$ must be characterized differently to ensure that the demand for bonds is well defined when capital is held. Define

$$S_B(G) = \{(R_1, \alpha) \mid (1-R_1)M^d(R_1, R_2) + (1-R_2)B^d(R_1, R_2) = G$$

$$R_2 = X > R_1^2 > 0, B^d(R_1, R_2) = \frac{\alpha \hat{NC}_3}{R_2} \quad \alpha \in [0, 1],$$

$$\langle M^d(R_1, R_2), B^d(R_1, R_2) \rangle > \langle 0, 0 \rangle\}.$$

Corollary

If

(a) $(R_1, \alpha) \in S_B(G)$ and

(b) $B(0) + M(1) > 0$,

then an equilibrium is given by the $\{p(t)\}$, $\{v(t)\}$, $\{M(t)\}$, $\{B(t)\}$ solutions to conditions (i) - (vi) of Proposition 1.

The proof is that the pair $(R_1, R_2) = (R_1, X)$ is included in $S_A(G)$, when α is given to make $B^d(R_1, R_2)$ well defined. Thus, the proof of Proposition 1 still applies.

When the assumptions of the corollary hold, it follows that

1. the supply of government bonds $\tilde{B}(t)$ can be computed as before using the constraint for open market operations,
2. the nominal interest rate is determined by the initial ratio of money to bonds discounted by α :

$$\frac{M(2)}{B(2)/\alpha} = \left(\frac{1}{1+r}\right) \frac{\beta}{\gamma},$$

3. the inflation rate is determined as before by the common growth rate of money and bonds, and
4. the endowment of bonds to the current middle-aged, $B(1)$, still is policy dependent to ensure $\frac{p(3)}{p(2)} = R_1$.

These conditions imply that when α is given, a stationary policy in terms of R_1 and R_2 can be associated with a stationary policy in terms of the ratio of money to bonds and the common growth rate of money to bonds. It is not true that a stationary policy in terms of R_1 and R_2 implies that α and, hence, the growth in bonds need be constant over time, however. A nonconstant $\alpha(t)$ and $B(t)$ growth path given by

$$\alpha(t+2) = \phi + R_2\alpha(t),$$

$$\phi \equiv \left[\frac{1+\beta+\gamma}{N\gamma\gamma} \right] [G - (1-R_1)M^d(R_1, R_2)], \text{ and}$$

$$\frac{B(t+2)}{B(t)} = (1+\Pi) \frac{\alpha(t+2)}{\alpha(t)}$$

are consistent with a given R_1 and $R_2 = X < 1$. Even in this case, though, $\alpha(t)$ approaches a limit $\frac{\phi}{1-R_2}$ and the two-period growth rate of bonds approaches the constant Π . Thus, there is little loss in generality in assuming that α is a constant from the outset.

IV. Properties of Feasible Policies Under Portfolio Autarky

In Section III it was shown that under any feasible, stationary policy, the growth rate of money is equal to the inflation rate,

$$\frac{M(t+2)}{M(t)} = 1 + \Pi,$$

and that a discrete-time Fisher effect obtains, $r = \rho + \Pi + \rho\Pi \approx \rho + \Pi$. In this section we want to examine the effects of alternative feasible policies on the initial price level $1/p(2)$, the rate of inflation Π , the nominal rate of interest r , and the welfare of all generations. The method is to take a given policy indexed by $(R_1, R_2) \in S_A(G)$ or $(R_1, \alpha) \in S_B(G)$. Then using Proposition 1 we ask how policy given by $M(t)$, $B(t)$, and $\tilde{B}(t)$ changes and how other variables of interest change under a new R'_2 such that $(R'_1, R'_2) \in S_A(G)$ or under a new α'' such that $(R''_1, \alpha'') \in S_B(G)$. Two numerical examples are examined first in order to provide insights about the qualitative results derived later using general methods.

Numerical Examples

Both examples assume the following parameter values:

$y = 1,000,$ endowment in first period of life
 $\beta = .9,$ second-period utility discount rate
 $\gamma = .6,$ third-period utility discount rate
 $N = 100,$ number of individuals in a generation
 $G = 10,000,$ real government deficit
 $M(1) = 36,000,$ endowment of money to current middle-aged
 $B(0) = 24,000,$ endowment of bonds to current old
 $K(0) = K(1) = 0,$ initial capital stocks.

In both examples we set $R_2 = X$, so that we have

$$M^d(R_1, R_2) = \frac{N\beta y}{1+\beta+\gamma} = 36,000 \text{ and}$$

$$B^d(R_1, R_2) = \frac{\alpha N \gamma y}{1+\beta+\gamma} = 24,000\alpha.$$

Example 1: $X = .9$

The government budget constraint in $S_B(G)$ yields:

$$R_1 = \frac{13}{18} + \frac{\alpha}{15}$$

so that for $\alpha \in [0,1]$, $R_1 > 0$ and $R_1^2 < X = R_2$. By letting α , the index of policy, vary, the model determines the following values:

Index	Policy Variables				Macro Variables		
	<u>B(1)</u>	<u>M(2)</u>	<u>B(2)</u>	<u>$\tilde{B}(2)$</u>	<u>Π</u>	<u>r</u>	<u>p(2)</u>
0.0	0	83,141	0	81,413	.918	.727	.433
0.2	11,441	70,175	15,569	72,505	.851	.666	.513
0.4	19,445	60,708	25,985	65,617	.783	.604	.593
0.6	25,252	53,492	33,122	60,235	.722	.550	.673
0.8	29,576	47,809	38,171	55,861	.665	.498	.753
1.0	32,856	43,217	41,635	52,071	.606	.446	.833

Some interesting results emerge from this table. First, suppose we start at $\alpha = 0$. Then feasible policies consist of (1) at time $t = 2$, decreasing the total amount of debt issued by the government and decreasing the amount of money held by the public, and (2) over all time, decreasing the rate of growth of money and bonds. The fact that total bonds sold on the open market must change implies that a change in α cannot be accomplished by open market operations alone. Second, note that as α increases, the ratio of privately held bonds to money rises and the nominal interest rate falls. Even though $\frac{B(2)}{M(2)}$ rises, $\frac{1}{\alpha} \frac{B(2)}{M(2)}$ —which is proportional to the interest rate—falls. Third, higher values of α result in lower inflation, lower interest rates, and a lower initial price level (the reciprocal of $p(2)$). This suggests that when $X < 1$, higher values of α benefit everyone, and this result is supported when we examine individual consumption under alternative policies:

Index	Current	Current		Current Young and		
	Old	Middle-Aged		All Future Generations		
α	$\hat{C}_3(0)$	$\hat{C}_2(1)$	$\hat{C}_3(1)$	\hat{C}_1	\hat{C}_2	\hat{C}_3
0.0	103.92	155.88	0	400.00	259.92	216.00
0.2	123.12	184.68	43.13	400.00	264.60	216.00
0.4	142.32	213.48	86.34	400.00	269.64	216.00
0.6	161.52	242.28	129.54	400.00	274.32	216.00
0.8	180.72	271.08	172.72	400.00	279.00	216.00
1.0	199.92	299.88	215.86	400.00	284.04	216.00

When $X < 1$, the real net rate of return on capital is negative. Welfare is increased by following monetary and fiscal policies which drive out capital.

Example 2: $X = 1.1$.

The government budget constraint in $S_B(G)$ yields:

$$R_1 = \frac{13}{18} - \frac{\alpha}{15},$$

so that for $\alpha \in [0,1]$, $R_1 > 0$ and $R_1^2 < R_2 = X$. Letting α vary, the model determines the following:

Index	Policy Variables				Macro Variables		
	<u>B(1)</u>	<u>M(2)</u>	<u>B(2)</u>	<u>$\tilde{B}(2)$</u>	<u>Π</u>	<u>r</u>	<u>p(2)</u>
0.0	0	83,141	0	99,468	.918	1.110	.433
0.2	14,510	70,175	20,474	95,249	.989	1.188	.513
0.4	25,588	60,708	36,759	92,871	1.064	1.271	.593
0.6	34,483	53,492	50,607	91,976	1.150	1.365	.673
0.8	41,913	47,809	62,664	91,691	1.234	1.458	.753
1.0	48,325	43,217	73,649	92,096	1.324	1.556	.833

Starting from $\alpha = 0$, feasible policies consist primarily of (1) open market sales at $t = 2$ (since $\tilde{B}(2)$ hardly changes), and (2) increasing the rates of growth of money and bonds over time. With a positive real rate of return on bonds, an increase in the amount of debt held in the private sector increases the real interest payments on the debt. In order to finance the initial deficit and the added interest expense, the government must resort to using more of the inflation tax. Since bonds cannot immediately be used for payments, an open market sale at $t = 2$ lowers the price level. This suggests that as α is increased through open market sales, the current old are made better off, while the current young and all future generations are made worse off. In this example, the current middle-aged are also made better off as α is increased:

Index	Current	Current		Current Young and		
	Old	Middle-Aged		All Future Generations		
α	$\hat{C}_3(0)$	$\hat{C}_2(1)$	$\hat{C}_3(1)$	\hat{C}_1	\hat{C}_2	\hat{C}_3
0.0	103.92	155.88	0	400.00	231.04	264.00
0.2	123.12	184.68	52.82	400.00	226.88	264.00
0.4	142.32	213.48	105.68	400.00	222.72	264.00
0.6	161.52	242.28	158.28	400.00	218.24	264.00
0.8	180.72	271.08	211.24	400.00	214.08	264.00
1.0	199.92	299.88	263.85	400.00	209.92	264.00

Thus, when the real rate of return on bonds is positive, a policy of increasing private holdings of bonds is not Pareto inefficient. It merely transfers resources to the current old and current middle-aged by taxing through inflation the young and all future generations.

The following proposition summarizes the set of feasible policies given by $(R_1, R_2) \in S_A(G)$ or $(R_1, \alpha) \in S_B(G)$ and the effects of alternative policies on macro variables. The effects are found by (a) letting R_2 or α vary, (b) using $S_A(G)$ or $S_B(G)$ to determine how R_1 must respond, and (c) differentiating the variables in conditions i - vi of Proposition 1 with respect to the change in policy.

Proposition 2

Let G be given.

A. Let $R_2 = X$. Then

Signs of Derivatives

		<u>X < 1</u>	<u>X = 1</u>	<u>X > 1</u>
Feasible Policy Trade-Off	$\frac{dR_1}{d\alpha}$	+	0	-
	$\frac{dB(1)}{d\alpha}$?	+	+
Feasible Monetary and Budget Policies	$\frac{dM(2)}{d\alpha}$	-	-	-
	$\frac{dB(2)}{d\alpha}$?	+	+
	$\frac{d\tilde{B}(2)}{d\alpha}$	-	-	?
Macro Policy Effects	$\frac{d\Pi}{d\alpha}$	-	0	+
	$\frac{dr}{d\alpha}$	-	0	+
	$\frac{dp(2)}{d\alpha}$	+	+	+

(The symbol '?' indicates that the derivative can be positive '+,' negative '-,' or zero '0,' depending on initial parameter values.)

B. Let $R_2 > X$. Then

Feasible Policy Trade-Off:

$$\frac{dR_1}{dR_2} < 0.$$

Feasible Monetary and Budget Policies:

$$\frac{dB(1)}{dR_2} > 0, \frac{dM(2)}{dR_2} = 0, \frac{dB(2)}{dR_2} > 0, \frac{d\tilde{B}(2)}{dR_2} > 0.$$

Macro Policy Effects:

$$\frac{d\Pi}{dR_2} > 0, \frac{dr}{dR_2} > 0, \frac{dp(2)}{dR_2} = 0.$$

Suppose that monetary and fiscal policies initially are on consistent courses. Now suppose that the government either raises its spending on public goods or lowers taxes so that the real budget deficit is larger for all time. Proposition 3 states that it is no longer feasible for the monetary authority to maintain the same course for the money stock.

Proposition 3

Let $\Psi(G) = \{[\langle \tilde{B}(t) \rangle, \langle M(t) \rangle], t=2, 3, \dots$ the sequences are jointly feasible for G and yield stationary r and $\Pi\}$. (By jointly feasible we mean that the implied values of all variables satisfy Proposition 1.) Let $[\langle \tilde{B}(t) \rangle^\circ, \langle M(t) \rangle^\circ]$ be arbitrary in $\Psi(G)$. Now suppose $G' \neq G$. Then there exists no sequence $\langle \tilde{B}(t) \rangle'$ such that $[\langle \tilde{B}(t) \rangle', \langle M(t) \rangle^\circ] \in \Psi(G')$.

Proof

For $M(t)$ to remain the same from $t = 2$ on, two conditions must be met:

- (1) $p(2)$ must remain the same, since $p(2)M(2)^\circ = \frac{\beta N y}{1+\beta+\gamma}$, and
- (2) R_1 must remain the same, since $\frac{M(t+1)^\circ}{M(t)^\circ} = \frac{1}{R_1}$, $t \geq 2$.

Thus, the proof consists of showing that the derivatives $\frac{dp(2)}{dG}$ and $\frac{dR_1}{dG}$ are constants for all G and not both can be zero.

If $R_2 > X$, we have $p(2) = \frac{Z_1 + Z_2 - G}{B(0) + M(1)}$, where

$$Z_1 = \frac{\beta Ny}{1 + \beta + \gamma} \text{ and } Z_2 = \frac{\gamma Ny}{1 + \beta + \gamma}.$$

Then $\frac{dp(2)}{dG} = \frac{-1}{B(0) + M(1)} < 0 \Rightarrow \frac{dM(2)}{dG} > 0$. In the case where $R_2 > X$, we have $G = p(2)[vB(2) - B(0) + M(2) - M(1)]$ or $G = Z_1 + Z_2 - p(2)(B(0) + M(1))$. Thus, the only way the government can increase the deficit G and still balance the budget constraint is to reduce the value of the initial endowments $B(0) + M(1)$. But with a lower value of money $p(2)$ and the same real amount demanded, more nominal money $M(2)$ must be supplied.

If $R_2 = X$, we have

a. $p(2) = \frac{Z_1 + \alpha Z_2 - G}{B(0) + M(1)}$ and

b. $R_1 = \frac{\alpha[\gamma(1-X)]}{\beta} + 1 - \frac{G(1+\beta+\gamma)}{\beta Ny}$ from $S_B(G)$.

From a

$$\frac{dp(2)}{dG} = 0 \Leftrightarrow \frac{d\alpha}{dG} = \frac{1+\beta+\gamma}{\gamma Ny}.$$

From b

$$\frac{dR_1}{dG} = \frac{\gamma(1-X)}{\beta} \frac{d\alpha}{dG} - \frac{(1+\beta+\gamma)}{\beta Ny}.$$

So, if $X = 1$

$$\frac{dR_1}{dG} = - \frac{(1+\beta+\gamma)}{\beta Ny} < 0 \Rightarrow \frac{d\pi}{dG} > 0$$

(a faster growth rate of money), and if $X > 0$ and $X \neq 1$

$$\frac{dR_1}{dG} = 0 \Leftrightarrow \frac{d\alpha}{dG} = \frac{1+\beta+\gamma}{(1-X)\gamma N\gamma}$$

and both conditions for $\frac{d\alpha}{dG}$ cannot be met. In the case where $R_2 = X$, it is possible to issue bonds at $t = 2$ to cover the increase in the government deficit while holding the price level constant. The additional debt outstanding, however, raises the path of real interest payments on the debt, so that the same common rate of growth for money and bonds will not satisfy the budget constraint. Alternatively, it is possible to issue money at $t = 2$ to cover the increase in the government deficit while holding the rate of inflation constant. The additional money raises the price level at $t = 2$ and thus taxes the endowments of the old and middle-aged.

We now turn to properties of optimal policies. The next proposition states that monetary and fiscal policies increase welfare by driving out capital when the real rate of return on capital is nonpositive.

Proposition 4

Suppose $X < 1$. Then any policy with $\alpha < 1$ is Pareto dominated by a policy with $\alpha' > \alpha$.

Proof

A. When $X < 1$ and $\alpha < 1$, it can be shown that:

$$1. \quad \frac{d\hat{C}_3(0)}{d\alpha} > 0$$

$$2. \quad \frac{d\hat{C}_2(1)}{d\alpha} > 0$$

$$3. \quad \frac{d\hat{C}_3(1)}{d\alpha} > 0$$

$$4. \quad \frac{d\hat{C}_1(t)}{d\alpha} = 0, \quad t \geq 2$$

$$5. \quad \frac{d\hat{C}_2(t)}{d\alpha} > 0, \quad t \geq 2$$

$$6. \quad \frac{d\hat{C}_3(t)}{d\alpha} = 0, \quad t \geq 2.$$

Inequalities 1, 2, 4, 5, and 6 follow directly from Proposition 2A given the individual demand functions for consumption. For inequality 3 to hold it must be shown that

$$\frac{d\hat{C}_3(1)}{d\alpha} = \frac{d(p(3)B(1))}{d\alpha} = \frac{d(R_1 p(2)B(1))}{d\alpha} > 0.$$

For arbitrary $X > 0$, the last derivative is simply

$$\frac{d(R_1 p(2)B(1))}{d\alpha} = \frac{X\gamma N_y}{1+\beta+\gamma} > 0.$$

B. When $X = 1$ and $\alpha < 1$, it can be shown that

$$1. \quad \frac{d\hat{C}_3(0)}{d\alpha} > 0$$

$$2. \quad \frac{d\hat{C}_2(1)}{d\alpha} > 0$$

$$3. \quad \frac{d\hat{C}_3(1)}{d\alpha} > 0$$

$$4. \quad \frac{d\hat{C}_1(t)}{d\alpha} = 0, \quad t \geq 2$$

$$5. \quad \frac{d\hat{C}_2(t)}{d\alpha} = 0, \quad t \geq 2$$

$$6. \quad \frac{d\hat{C}_3(t)}{d\alpha} = 0, \quad t \geq 2.$$

Again inequalities 1, 2, 4, 5, and 6 follow directly from Proposition 2A given the individual demand functions for consumption. The proof in part A of this proposition that inequality 3 holds still applies.

The next proposition states that if the real rate of return on capital is positive, monetary and fiscal policies which increase the private sector's real holdings of bonds cause changes in intergenerational income distributions which are Pareto noncomparable.

Proposition 5

Let G be given and suppose $R_2 = X > 1$. Let policy A be identified by $\alpha < 1$ and let policy B be identified by $\alpha' > \alpha$. Then policy B as compared to policy A makes the current old and the current middle-aged better off, but makes the current young and all future generations worse off.

Proof

In the proof of Proposition 4 it followed that for arbitrary $X > 0$,

$$\frac{d\hat{C}_3(0)}{d\alpha} > 0, \frac{d\hat{C}_2(1)}{d\alpha} > 0, \frac{d\hat{C}_3(1)}{d\alpha} > 0, \frac{d\hat{C}_1(t)}{d\alpha} = \frac{d\hat{C}_3(t)}{d\alpha} = 0, t \geq 2.$$

By Proposition 2A when $X > 1$, $\frac{dR_1}{d\alpha} < 0$, which implies $\frac{d\hat{C}_2(t)}{d\alpha} < 0, t \geq 2$.

When the rate of return on capital is positive, it still follows that increasing individuals' real holdings of bonds at $t = 2$ lowers the price level at $t = 2$. This, then, makes the endowments of the current old and current middle-aged more valuable. Since bonds now yield a negative real return to the government, even more of the inflation tax is required to satisfy the budget constraint. A higher rate of inflation means a higher tax on real money holdings, and this makes the current young and all future generations worse off.

The final proposition in this section examines the welfare implications of alternative policies when all capital has been driven out of the system.

Proposition 6

Let G be given, suppose $R_2 = X$ and $\alpha = 1$ or $R_2 > X$, and index alternative feasible policies by R_2 . Then,

- A. If $R_2 < R_1$, a policy with $R_2 = R_1$ Pareto dominates.
- B. If $R_2 > R_1$, a policy which increases R_2 further does not affect the welfare of the current old, increases the welfare of the current middle-aged, and decreases the welfare of the current young and all future generations.

Proof

Note that $R_2 < R_1$ is possible only if $R_2 < 1$, since $R_2 > R_1^2$ is required.

The proofs of parts A and B of the proposition are developed together. By Proposition 2B we have $\frac{dp(2)}{dR_2} = 0$, and this implies

$$\frac{\hat{d}C_3(0)}{dR_2} = \frac{\hat{d}C_2(1)}{dR_2} = 0.$$

Next, using the fact that

$$N \frac{\hat{d}C_3(1)}{dR_2} = \frac{d(R_1 p(2) B(1))}{dR_2} = p(2) \left[R_1 \frac{dB(1)}{dR_2} + B(1) \frac{dR_1}{dR_2} \right],$$

it is straightforward to show that $\frac{\hat{d}C_3(1)}{dR_2} > 0$. For $t \geq 2$ a representative utility function is given by $U(C_1, C_2, C_3) = \ln C_1 + \beta \ln C_2 + \gamma \ln C_3$, so that

$$\frac{dU}{dR_2} = \frac{1}{\hat{C}_1} \frac{d\hat{C}_1}{dR_2} + \frac{\beta}{\hat{C}_2} \frac{d\hat{C}_2}{dR_1} \frac{dR_1}{dR_2} + \frac{\gamma}{\hat{C}_3} \frac{d\hat{C}_3}{dR_2}.$$

With

$$\hat{C}_1 = \frac{\gamma}{1+\beta+\gamma}, \quad \hat{C}_2 = \frac{\beta\gamma R_1}{1+\beta+\gamma}, \quad \hat{C}_3 = \frac{\gamma\gamma R_2}{1+\beta+\gamma}, \quad \text{and} \quad \frac{dR_1}{dR_2} = -\frac{\gamma}{\beta}$$

by $S_A(g)$, we have

$$\frac{dU}{dR_2} = \frac{\gamma(R_1 - R_2)}{R_1 R_2},$$

and that completes the proof. Note, though, that if $R_2 > R_1$ a further increase in R_2 increases the welfare of only the current middle-aged, and it does this because it boosts their endowment of bonds $B(1)$ required for stationarity. If we discount this result which seems due to a mathematical convenience, the model would imply that R_2 should not be pushed above R_1 , when $R_2 > X$.

V. Laissez-Faire

Under laissez-faire all exchanges are permitted and are costless. In the laissez-faire regime it is feasible for the current young to lend to the current middle-aged with the loan being paid back in the next period with the proceeds from maturing government bonds or maturing capital. There will be no within-generation exchanges, because individuals of the same generation have identical endowments and tastes. There will be no exchanges between the current young and the current old, because the old desire only to borrow but will not be alive in the next period to pay off the loans.

Laissez-faire forces the two-period loan rate to be the same as what could be earned on two consecutive one-period loans. The next proposition states that when there is no capital, laissez-faire can be considered a special case of portfolio autarky with this equality of rates condition imposed.

Define R_1' to be the gross rate of return on one-period loans, and let $M_i^d(\cdot, \cdot)$, $B_i^d(\cdot, \cdot)$ be the real aggregate demand functions for money and bonds, respectively, under regime $i = PA$ (portfolio autarky) or $i = LF$ (laissez-faire).

Proposition 7

Suppose $G > 0$, $R_2 > X$, and $R_2 > R_1^2$. Then,

(a) $(R_1')^2 = R_2$.

(b) Let (R_1, R_2) be an equilibrium under laissez-faire. Then, $(R_1', R_2) \in S_A(G)$; in particular, individual budget constraints are $y = C_1 + C_2/R_1' + C_3/R_2$ and government revenue is $(1-R_1)M_{LF}^d(R_1, R_2) + (1-R_2)B_{LF}^d(R_1, R_2) = (1-R_1')M_{PA}^d(R_1', R_2) + (1-R_2)B_{PA}^d(R_1', R_2)$.

The proof is in the appendix.

Proposition 7 states that when $R_2 > X$, a laissez-faire economy can be considered a portfolio autarky economy with $R_1^2 = R_2$. Suppose now that $R_2 = X$. If $X > 1$, G cannot be financed under laissez-faire, since with $R_2 = X$ the maximum revenue which can be raised is zero. By Proposition 7,

$$\begin{aligned} (1-R_1)M_{LF}^d(R_1, X) + (1-R_2)B_{LF}^d(R_1, X) &= \\ (1-\sqrt{X})M_{PA}^d(\sqrt{X}, X) + (1-X)B_{PA}^d(\sqrt{X}, X) &= \begin{cases} 0 & X=1 \\ & \\ <0 & X>1 \end{cases} \end{aligned}$$

Suppose, instead, that $X < 1$. Then with $R_2 = X$, we know that an optimal policy requires that capital be driven out (see Proposition 4).

These considerations lead us to the following two propositions.

Proposition 8

Let G_{LF} and G_{PA} be the least upper bounds for deficits under laissez-faire and portfolio autarky, respectively. Then $X > 0 \Rightarrow G_{LF} < G_{PA}$.

Proof

Suppose first that $X > 1$. Then $G_{LF} = 0$, but

$$G_{PA} = M_{PA}^d(0,0) = \frac{\beta Ny}{1+\beta+\gamma}.$$

Suppose, instead, that $X < 1$. Then

$$\begin{aligned} G_{LF} &= (1-\sqrt{X})M_{PA}^d(\sqrt{X},X) + (1-X)B_{PA}^d(\sqrt{X},X) = \\ &= (1-\sqrt{X})\frac{\beta Ny}{1+\beta+\gamma} + (1-X)\frac{\gamma Ny}{1+\beta+\gamma} \end{aligned}$$

$$\begin{aligned} G_{PA} &= M_{PA}^d(0,X) + (1-X)B_{PA}^d(0,X) = \\ &\frac{\beta Ny}{1+\beta+\gamma} + (1-X)\frac{\gamma Ny}{1+\beta+\gamma}. \end{aligned}$$

Then $X > 0 \Rightarrow G_{PA} > G_{LF}$.

Proposition 9

Let $\hat{\Psi}(G)_i = \{[\langle \tilde{B}(t), M(t) \rangle], t=2, 3, \dots\}$ the pair of sequences is in $\Psi(G)$ (see Proposition 3) under regime i and is Pareto optimal under regime i where $i = PA$ or $i = LF$. Let G and X be given, but arbitrary, so that $\hat{\Psi}(G)_{LF} \neq \phi$. Then, there exists a policy in $\hat{\Psi}(G)_{PA}$ that Pareto dominates any policy in $\hat{\Psi}(G)_{LF}$.

Proof

By Propositions 7 and 8, a policy in $\hat{\Psi}(G)_{LF}$ can be associated with a policy under portfolio autarky which produces an (R_1, R_2) pair such that

$$R_1^2 = R_2 = X < 1 \quad (\alpha=1) \text{ and}$$

$$(1-R_1)M_{PA}^d(R_1, R_2) + (1-R_2)B_{PA}^d(R_1, R_2) = G.$$

But from $R_1^2 = R_2 < 1$, it follows that $R_1 > R_2$, and by Proposition 6, it follows that there exists a PA policy with $R_2 > X$ which Pareto dominates.

VI. Relevance of the Model and Its Implications

Because the structure of the model is so specialized, some care must be exercised in applying its conclusions to current policy issues. The modeling strategy was to adopt a simple, mathematically tractable structure in order to provide counterexamples, insights, and inferences on the policy mix question.^{3/} The burden is to argue that the important conclusions will hold under more general formulations.

Two important conclusions are that monetary and fiscal policies must be coordinated and that rules which limit the liquidity of government bonds may be desirable. These conclusions do not seem to depend on the overlapping generations structure of the model or on the special assumptions of three-period lived agents, linear technology, log-linear utility, or y-0-0 endowment pattern.^{4/}

The need to coordinate policies takes two forms: the need to coordinate money and bond issue for a given deficit and the need to coordinate

^{3/}A defense of the use of simple models is given in Cass-Shell.

^{4/}See Bryant and see Wallace for defenses of the overlapping generations model.

the path of money with the level of deficits. The first need follows from the government's real budget constraint. The government's real deficit net of interest must be financed by the sum of its real taxes on private holdings of money and bonds. Since a change in either money or bond issue generally changes the sum, it follows that for a given deficit the issues of money and bonds must be coordinated.

The second need follows from welfare considerations. One consideration is that only policies which yield stationary solutions should be objects of choice. Individual's utilities depend on the real rates of return they face over their lifetimes. Policies which yield nonstationary solutions imply varying real rates of return over time and, thus, cause arbitrary differences in the welfare of individuals from different generations. Stationary solutions suggest equal treatment of agents of different generations. With respect to this restricted class of policies, another consideration is that if an initial equilibrium is optimal then it cannot be possible to finance an increase in the deficit by bond issue alone. For bond issue to yield revenue to the government in a stationary equilibrium, the real net rate of return on bonds must be negative. If it is negative, then the initial equilibrium can be optimal only if all capital is crowded out. But then no additional deficits can be financed by bond issue alone. In terms of the notation of this paper, the argument is that additional G can be financed by additional \tilde{B} alone, only if in the initial equilibrium $\rho < 0$ and $\alpha < 1$. But if $\rho < 0$, $\alpha < 1$ cannot be optimal.

The desirability of placing restrictions on the use of bonds rests on the ability of the government to tax discriminate. If the restrictions separate money and bond holders into two classes of individuals characterized by different preferences, then they permit the government to tax each class at different rates. The two taxes, used to finance given deficits, are deter-

mined by policy mix. By appropriate choice of the two rates, the government can increase welfare compared to the case where it has a single tax.

Assuming these conclusions are general, they have direct relevance to current issues on policy mix and financial structure. These issues are discussed in turn below.

Monetarists call for a policy of reducing growth in money in order to lower the rate of inflation. Various interpretations are possible of what this prescription assumes about the relationship between monetary and fiscal policies. One is that the optimal monetary policy can be made independently of fiscal policy. In this case the optimal path of money would be the same no matter what the course for fiscal policy. A second is that the optimal monetary policy appropriately can offset the inflationary impact of a given fiscal policy. In this case the optimal path of money would be lower the higher the levels of future budget deficits. A third interpretation is that there can be only a small range of government budget policies which are consistent with the prescribed monetary policy. In this case a reduction in the growth of money would force a tightening in budget policies.

The implication from this paper is that only the third interpretation is reasonable. A monetary policy can neither disregard fiscal policy nor offset it. If, however, monetary policy were to stick to a tight course, fiscal policy would either have to follow a correspondingly tight course or the government would be forced into insolvency.

The other implication is that the continued relaxation of restrictions on financial intermediary portfolios may not be desirable. In recent years these restrictions have been relaxed both by deliberate government deregulation and by new innovations introduced by private firms to circumvent existing regulations. Some of the freeing up of the financial industry has

the effect of making government bonds more liquid. One example is that money market funds can buy Treasury securities directly, or indirectly through bank CDs, and use them as backing for customer checking accounts. This freeing up of the financial industry should then result in more inflation for a given deficit policy, since it increases the liquidity of government debt instruments. Moreover, it may decrease economic efficiency by limiting the government to a single debt instrument.

Appendix

(a) The individual's steady-state maximization problem under laissez-faire is

$$\max \ln C_1(t) + \beta \ln C_2(t) + \gamma \ln C_3(t)$$

subject to

$$C_1(t) = y - p(t)wb^d(t-1) - p(t)vb^d(t) - p(t)m(t)$$

$$C_2(t) = p(t+1)b^d(t-1) + p(t+1)wb^s(t) + p(t+1)m(t)$$

$$C_3(t) = p(t+2)[b^d(t) - b^s(t)],$$

where

w = price of one-period-old bonds,

$b^d(t-1)$ = bonds issued in period $t-1$ which are demanded by individuals in $\{N(t)\}$,

$b^d(t)$ = bonds issued in period t which are demanded by individuals in $\{N(t)\}$, and

$b^s(t)$ = bonds issued in period t , and purchased by individuals in $\{N(t)\}$ to be sold in period $t+1$, and necessarily $b^s(t) \leq b^d(t)$.

Define

$$I_1 = p(t)wb^d(t-1) + p(t)vb^s(t) + p(t)m(t)$$

$$I_2 = p(t)v(b^d(t) - b^s(t)), \text{ and}$$

$$R'_1 = \frac{p(t+1)}{p(t)w}.$$

We then have

$$(i) \quad C_1(t) = y - I_1 - I_2,$$

$$(ii) \quad C_2(t) = \left[\frac{p(t+1)}{p(t)w} \right] p(t) w b^d(t-1) + \\ \left[\frac{p(t+1)w}{p(t)v} \right] p(t) v b^s(t) + \left[\frac{p(t+1)}{p(t)} \right] p(t) m(t), \text{ and}$$

$$(iii) \quad C_3(t) = \left[\frac{p(t+2)}{p(t)v} \right] [p(t)v(b^d(t)-b^s(t))].$$

The individual can invest in one-period bonds for second-period consumption, either by buying one-period-old bonds at a price w and selling them at par in the next period or by buying new issues at a price v and selling them next period at a price w . Since these two investments must yield the same rate of return in equilibrium, we have

$$R'_1 \equiv \frac{p(t+1)}{p(t)w} = \frac{p(t+1)w}{p(t)v} \Rightarrow v = w^2.$$

But then,

$$(R'_1)^2 = \left[\frac{p(t+2)}{p(t+1)w} \right] \left[\frac{p(t+1)}{p(t)w} \right] = \frac{p(t+2)}{p(t)w^2} = \frac{p(t+2)}{p(t)v} = R_2.$$

(b) We have two cases to examine:

$$(i) \quad R_1 < R'_1 \text{ and}$$

$$(ii) \quad R_1 = R'_1.$$

In either case under portfolio autarky,

$$M_{PA}^d(R'_1, R_2) = \frac{\beta Ny}{1+\beta+\gamma} \text{ and}$$

$$B_{PA}^d(R'_1, R_2) = \frac{\gamma Ny}{1+\beta+\gamma}.$$

We then have

$$(1-R'_1)M_{PA}^d(R'_1, R_2) + (1-R_2)B_{PA}^d(R'_1, R_2) = \frac{(1-R'_1)Ny}{1+\beta+\gamma}(\beta+(1+R'_1)\gamma).$$

If $R_1 < R'_1$, the demand for money under laissez-faire is zero, since one-period bonds dominate. The steady-state individual budget constraint then is

$$y = C_1 - C_2/R'_1 - C_3/R_2.$$

What must be shown is that

$$(1-R_2)B_{LF}^d(R_1, R_2) = (1-R'_1)M_{PA}^d(R'_1, R_2) + (1-R_2)B_{PA}^d(R'_1, R_2).$$

Under laissez-faire, in steady-state we must have

$$p(t)wb^d(t-1) = p(t+1)wb^s(t) \text{ or } b^d(t-1) = \frac{p(t+1)}{p(t)}b^s(t),$$

the real demand for one-period bonds by $h \in \{N(t)\}$ must be equal to h 's real supply of one-period bonds next period. Thus, $B_{LF}^d(R_1, R_2) \equiv (Np(t)vb^d(t))$ is found by

$$p(t)vb^d(t) = \hat{C}_3/R_2 + p(t)vb^s(t) \quad \text{using aiii of this proof}$$

$$p(t)vb^s(t) = \hat{C}_2/R_1 - p(t)wb^d(t-1) \quad \text{using aii of this proof}$$

$$= \hat{C}_2/R_1 - p(t)w \frac{p(t+1)}{p(t)}b^s(t) \quad \text{using steady-state condition.}$$

So

$$p(t)vb^s(t) \left[1 + \frac{p(t+1)}{p(t)w} \right] = \hat{C}_2/R'_1$$

with $w^2 = v$ or

$$p(t)vb^s(t) = \hat{C}_2[R'_1(1+R'_1)].$$

We then have

$$B_{LF}^d(R_1, R_2) \equiv Np(t)vb^d(t) = \hat{NC}_2/[R'_1(1+R'_1)] + \hat{NC}_3/(R'_1)^2 \text{ and}$$

$$(1-R_2)B_{LF}^d(R_1, R_2) = (1-(R'_1)^2)B_{LF}^d(R_1, R_2) =$$

$$\hat{NC}_2\left[\frac{1-R'_1}{R'_1}\right] + \hat{NC}_3\left[\frac{1-(R'_1)^2}{(R'_1)^2}\right] =$$

$$\left[\frac{1-R'_1}{R'_1}\right] \left[\frac{\beta Ny}{1+\beta+\gamma}\right]R'_1 + \left[\frac{1-(R'_1)^2}{(R'_1)^2}\right] \left[\frac{\gamma Ny}{1+\beta+\gamma}\right](R'_1)^2 =$$

$$\left[\frac{Ny(1-R'_1)}{1+\beta+\gamma}\right] [\beta+(1+R'_1)\gamma].$$

If, on the other hand, $R_1 = R'_1$, individuals are indifferent between holding money and holding one-period bonds for second-period consumption. Again, the steady-state individual budget constraint is $y = C_1 + C_2/R'_1 + C_3/R_2$. What must be derived is government revenue under laissez-faire.

Let δ be the proportion of second-period consumption financed by real money balances. Then, using aii and aiii of this proof we have,

$$(i) \quad p(t)vb^d(t) = \hat{C}_3/R_2 + p(t)vb^s(t)$$

$$(ii) \quad p(t)vb^s(t) = (1-\delta)\hat{C}_2/R'_1 - p(t)wb^d(t-1)$$

$$(iii) \quad p(t)m^d(t) = \delta\hat{C}_2/R'_1$$

and assuming stationarity

$$(iv) \quad b^d(t-1) = \frac{p(t+1)}{p(t)}b^s(t).$$

Substituting (iv) into (ii) and solving we have

$$p(t)vb^d(t) = (1-\delta)\hat{C}_2/[R_1'(1+R_1')]$$

and inserting this expression into (i) yields

$$p(t)vb^d(t) = (1-\delta)\hat{C}_2/[R_1'(1+R_1')] + \hat{C}_3/(R_1')^2.$$

We then can write when $R_1 = R_1'$,

$$M_{LF}^d(R_1, R_2) \equiv Np(t)m^d(t) = \delta NC_2/R_1 = \frac{\delta N\beta y}{1+\beta+\gamma} \text{ and}$$

$$B_{LF}^d(R_1, R_2) \equiv Np(t)vb^d(t) = N(1-\delta)\hat{C}_2/[R_1(1+R_1)] + NC_3/R_1^2 =$$

$$\frac{Ny}{1+\beta+\gamma} [(1-\delta)\beta/(1+R_1)+\gamma].$$

So

$$(1-R_1)M_{LF}^d(R_1, R_2) + (1-R_2)B_{LF}^d(R_1, R_2) =$$

$$(1-R_1)\frac{\delta N\beta y}{1+\beta+\gamma} + (1-R_1)(1-\delta)\frac{N\beta y}{1+\beta+\gamma} + (1-R_1^2)\frac{\gamma Ny}{1+\beta+\gamma} =$$

$$\left[\frac{(1-R_1)Ny}{1+\beta+\gamma}\right] [\beta+(1+R_1)\gamma].$$

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