A Hybrid Fiat-Commodity Monetary System

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In this paper I describe a "monetary" system in which backing is provided for the government's liabilities by way of contingent resort to taxes. The system has some of the features of a commodity money system with a large seignorage spread between bid and ask prices. It is studied within the context of a one-good, pure exchange model of two-period-lived overlapping generations in which, aside from various uniform boundedness assumptions, considerable diversity is allowed both within and across generations. Two results are established: (i) the existence of at least one perfect foresight competitive equilibrium, and (ii) the Pareto optimality of <u>any</u> such equilibrium.

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In this paper I describe a "monetary" system in which backing is provided for the government's liabilities by way of contingent resort to taxes. The system is studied within the context of a one-good, pure exchange model of two-period-lived overlapping generations in which, aside from various uniform boundedness assumptions, considerable diversity is allowed both within and across generations. Two results are established: (i) the existence of at least one perfect foresight competitive equilibrium, and (ii) the Pareto optimality of <u>any</u> such equilibrium.

The (monetary) system studied has some of the features of a commodity money system with a large seignorage spread between bid and ask prices. The system is described in terms of three parameters, $(\underline{p}, \overline{p})$ and M, and a supporting tax-transfer scheme, where $\underline{p}$ and $\overline{p}$ are positive prices of money--somewhat like bid and ask prices, respectively--and M is some positive number of units of money. The tax-transfer scheme aside, government action at any date t takes the form of the supply curve (correspondence) shown in Figure 1; M(t) is the outstanding stock of money held by the public from t to t+1 and p(t) is the market price of a unit of money at t in units of time t goods (the inverse of the price level at t). I assume that this scheme is set up at t = 1 and that M(0) = 0. Feasibility of the scheme is guaranteed by appropriate choices of $\underline{p}$, $\overline{p}$, and M. I call this scheme a hybrid fiat-commodity scheme because with $\underline{p} = 0$ and $\overline{p}$ sufficiently large it is a fiat scheme with a fixed supply of fiat money M (hereafter called a pure fiat scheme), while with $\underline{p} = \overline{p} > 0$ it is a commodity scheme with no seignorage spread.

[INSERT FIGURE 1]

In one important sense, the hybrid scheme is a mixture of a pure fiat scheme and a pure tax-transfer scheme (a social security scheme); it makes some use of market pricing of net government indebtedness and some use of taxes and transfers. Its one advantage vis-a-vis the pure fiat scheme is that any equilibrium under the hybrid scheme is Pareto optimal. For the pure fiat scheme one can at best demonstrate that there exists an optimal equilibrium; it is well known that there can exist nonoptimal equilibria under the pure fiat scheme. The presumed advantage of the hybrid scheme vis-a-vis the pure tax-transfer scheme is its lesser use of taxes and transfers. I say presumed because the framework I set out is not rich enough to imply that less use of taxes and transfers is preferable to more use of taxes and transfers.

The rest of this paper is organized as follows. In Section 1, I describe the economy studied. In Section 2, I describe the conditions for perfect foresight competitive equilibrium, establish existence of such an equilibrium, and--using a Balasko-Shell (1980) optimality criterion--prove that any such equilibrium is Pareto optimal. Two examples are provided in Section 3 and concluding remarks in Section 4.

1. The Economy

1.1 Endowments and Preferences

Time is discrete and takes on the values $t = 1, 2, \ldots$. At each date t, $t \geq 1$, a new generation, generation t, of size $N(t)$ appears and is present in the economy at t and $t+1$. The $N(0)$ members of generation 0 are present only at $t = 1$. There is a single, nonproduced good in the economy at each date. Society's endowment of this good, denoted $W(t)$, is bounded; i.e., $W(t) \leq W$ for all $t \geq 1$.

Let $w_i^j(t) \geq 0$ be the endowment of time t good of member j of generation i and let $W_t(t) = \sum_j w_t^j(t)$ and $W_t(t+1) = \sum_j w_t^j(t+1)$, where summation over j is summation over the members of generation t. Here $W_t(t)$ ($W_t(t+1)$) is the total amount of time t ($t+1$) good owned by members of generation t. I assume $W_{t-1}(t) + W_t(t) = W(t)$, $W_t(t) \geq W_1 > 0$ and $W_t(t+1) \geq W_2 > 0$ for all $t \geq 1$; that is, individual endowments of time t good for members of generations $t-1$ and t exhaust society's endowment, $W(t)$, and the aggregate endowment of generation t of both time t and time $t+1$ goods is bounded away from zero. I also assume that individual endowments are uniformly bounded away from zero; i.e., there exists $w > 0$ such that $(w_t^j t), w_t^j(t+1)) \geq (w,w)$ for all j and $t \geq 1$.

As regards preferences, each member of generation 0 maximizes consumption of time 1 good, while each member j of generation t, $t > 0$, has a twice differentiable utility function $u_t^j(\cdot,\cdot)$ that is defined on positive consumption of time t and time $t+1$ goods, respectively. The function u_t^j has positive first-order partial derivatives and displays a diminishing marginal rate of substitution. Moreover, if (x,y) is such that $(x,y) \leq (W,W)$ and $u_t^j(x,y) \geq u_t^j(w_t^j(t), w_t^j(t+1)/2)$ for some j and $t > 0$, then it is assumed (a) that $x > 0$ and $y > 0$, (b) that there exist scalars $h > 0$ and $H > 0$ (not dependent on j or t) such that $h \leq u_{t1}^j(x,y)/u_{t2}^j(x,y) \leq H$ (i.e., bounded marginal rates of substitution), and (c) that there exists another pair of scalars (not dependent on j or t) that provide a uniform and positive lower bound and a uniform and positive upper bound on the Gaussian curvature of any indifference curve of u_t^j at any such (x,y).

1.2 The "Monetary" System

I assume that the parameters $(\underline{p}, \overline{p})$ and M satisfy $M > 0$ and $0 < \underline{p}M \leq W_2/2 < W \leq \overline{p}M$. Note that the last inequality implies $p(t)M(t) < \overline{p}M$ for all t and, hence, that no money is ever purchased from the government at $\overline{p}$. Therefore, $M(t) \leq M$ for all t.

At time t, the total of government expenditures under this scheme is $p(t)[M(t-1)-M(t)]$, which I denote by $\tau(t)$. Note that if $\tau(t) > 0$, then $M(t) < M$ and $p(t) = \underline{p}$. It follows that $\tau(t) \leq \underline{p}M$. I assume that $\tau(t)$ is financed by lump-sum taxes as follows.

All taxes (and transfers) are levied on the old. Thus, let $\tau_t^j(t+1)$ be the tax payable at t+1 by member j of generation t. I assume that $\tau_t^j(t+1) \leq w_t^j(t+1)/2$, that $\tau_t^j(t+1)$ is a continuous function (possibly dependent on j and t) of $\tau(t+1)$ and, of course, that $\sum_j \tau_t^j(t+1) = \tau(t+1)$. That the first and third of these assumptions can be satisfied follows from $\tau(t+1) \leq \underline{p}M \leq W_2/2 \leq \sum_j w_t^j(t+1)/2$. The second assumption, continuity, is important for the existence-of-equilibrium proof. All three assumptions are satisfied by the following "proportional" tax scheme: $\tau_t^j(t+1) = w_t^j(t+1)\tau(t+1)/W_t(t+1)$.

2. Perfect Foresight Competitive Equilibrium: Existence and Optimality

From now on, unless otherwise noted, I will use the term equilibrium to refer to a perfect foresight competitive equilibrium.

I find it convenient to state equilibrium conditions in terms of an aggregate saving function for members of generation t. Thus, for member j of generation t, let $s_t^j(1+r(t),\tau_{t+1}^j)$ be the unique and continuous solution for $w_t^j(t) - c_t^j(t)$ to the problem: choose $(c_t^j(t),c_t^j(t+1))$ to maximize $u_t^j(\cdot,\cdot)$ subject to

$$(1) \qquad c_t^j(t) + c_t^j(t+1)/[1+r(t)] \leq w_t^j(t) + [w_t^j(t+1)-\tau_{t+1}^j]/[1+r(t)].$$

Then, let

$$S_t[1+r(t),\tau(t+1)] = \textstyle\sum_j s_t^j(1+r(t),\tau_{t+1}^j).$$

So defined, the function S_t, the aggregate saving function for generation t, is well defined and continuous in each of its arguments.

The first result we need is one that relates equilibrium values of 1+r(t) to p(t) and p(t+1), where, recall, p(t) is the price at t in terms of time t good of a unit of money. In equilibrium, $p(t+1) \geq \underline{p}$ and

$$(2) \qquad 1+r(t)\begin{cases} = p(t+1)/p(t) & \text{if } M(t) > 0 \\ \geq p(t+1)/\underline{p} & \text{if } M(t) = 0. \end{cases}$$

The first line of (2) is an arbitrage condition. It says that if money is held from t to t+1, then the terms of trade between time t and time t+1 goods (the time t interest rate) implied by the values of money at t and t+1 must be the market interest rate. As for the second line, it gives a lower bound on the market interest rate if no money is purchased by the young at t. In particular, it implies that if M(t) = 0, then $r(t) \geq 0$ because money is available to be purchased at $\underline{p}$ and then can be sold at $p(t+1) \geq \underline{p}$.

Now let $q(t) \equiv p(t)M(t)$ (real money holdings held from t to t+1 by generation t), let $q \equiv \underline{p}M$ and let R, a function on $[0,W] \times [0,W]$ be defined as follows:

$$
(3) \qquad R(x,y) = \begin{cases} 1 & \text{if } (0,0) \leq (x,y) \leq (q,q) \\ y/x & \text{if } (q,q) \leq (x,y) \leq (W,W) \\ y/q & \text{if } (0,q) \leq (x,y) \leq (q,W) \\ q/x & \text{if } (q,0) \leq (x,y) \leq (W,q). \end{cases}
$$

So defined, R is continuous and, by (2), relates equilibrium values of $r(t)$, $q(t)$, and $q(t+1)$ as follows: $1+r(t) \geq R(q(t),q(t+1))$ and $1+r(t) = R(q(t),q(t+1))$ if $q(t) > 0$.

We can also express the equilibrium value of $\tau(t+1)$ in terms of $q(t)$, $q(t+1)$, and $r(t)$. First, from the definition of $q(t)$, $\tau(t+1) = q(t)p(t+1)/p(t) - q(t+1)$. Then, since in equilibrium $p(t+1)/p(t) = 1+r(t)$ except when $M(t)$ and, hence, $q(t)$ is zero, in equilibrium $\tau(t+1) = q(t)[1+r(t)] - q(t+1)$.

Therefore, we have the following:

<u>Definition</u>: An equilibrium is a nonnegative $q(t)$ sequence and a positive $1+r(t)$ sequence that for all $t \geq 1$ satisfies

$$
(4) \qquad S_t[1+r(t),q(t)[1+r(t)]-q(t+1)] = q(t)
$$

$$
(5) \qquad [1+r(t)] - R[q(t),q(t+1)] \geq 0
$$

$$
(6) \qquad \{1+r(t)-R[q(t),q(t+1)]\}q(t) = 0.
$$

Equation (4) says that aggregate saving must equal the value of money, while conditions (5) and (6) relate the market rate of interest to $q(t)$ and $q(t+1)$ as implied by (2).

We now state and prove our two results.

Proposition 1: An equilibrium exists.

Proof: The first step is to show that for any $q(t+1) \in [0,W]$, there exists at least one pair $[q(t), 1+r(t)] \in [0,W] \times (0,\infty)$ that satisfies (4)-(6).

Let $\emptyset_t(x,y) = S_t[R(x,y), xR(x,y)-y]$. So defined, for any fixed $y \in [0,W]$, $\emptyset_t(x,y)$ is defined and continuous for all $x \in [0,W]$ and $\emptyset_t(x,y) < W$. There are two cases to consider. If $\emptyset_t(0,y) \geq 0$, then there exists at least one value of $x \in [0,W]$ with $\emptyset(x,y) = x$. At any such x and $1+r(t) = R(x,y)$, (4)-(6) are satisfied. If $\emptyset_t(0,y) < 0$ (desired saving is negative at $1+r(t) = R(0,y)$), then, by the continuity of S_t and by our endowment and preference assumptions, there exists some $1+r(t) > R(0,y)$ such that $S_t[1+r(t),-y] = 0$. This value of $1+r(t)$ and $q(t) = 0$ satisfy (4)-(6).

This establishes the existence of a mapping Ψ_t that associates with each $q(t+1) \in [0,W]$ a nonempty subset of $[0,W] \times (0,\infty)$ with the property that $q(t+1)$ and $(q(t), 1+r(t)) \in \Psi_t(q(t+1))$ satisfy (4)-(6). Let us denote by $\gamma_t(q(t+1))$ the projection of $\Psi_t(q(t+1))$ on $[0,W]$. It follows from the continuity of $\emptyset_t(x,y) - x$ in (x,y) and from the continuity of $S_t(1+r(t),-y)$ in its arguments that γ_t maps nonempty compact subsets of $[0,W]$ into nonempty compact subsets of $[0,W]$.

Now let $I = [0,W]$ and denote by $\gamma_t(B)$ the range of γ_t on the set B. Also, let

$$Q_0 = X_1^\infty I$$

$$Q_1 = \gamma_1(I) \times X_2^\infty I$$

$$Q_2 = \gamma_1(\gamma_2(I)) \times \gamma_2(I) \times X_3^\infty I$$

$$\vdots$$

$$Q_t = \gamma_1(\gamma_2(..\gamma_t(I))...) \times \gamma_2(...(\gamma_t(I))...) \times ... \times \gamma_t(I) \times X_{t+1}^\infty I$$

$$\vdots$$

So defined, each Q_t is an infinite product of compact sets and, therefore, is compact. Moreover, by the property of γ_t established above, each Q_t is nonempty. Then since $Q_t \supset Q_{t+1}$ for all $t \geq 0$, it follows that $Q \equiv \bigcap_0^\infty Q_t$ is nonempty and that any element $\{q(t)\} \in Q$ satisfies $q(t) \in \gamma_t(q(t+1))$ for all $t \geq 1$. It is obvious from the first step in the proof that we can associate with any such $q(t)$ sequence a positive $1+r(t)$ sequence such that both satisfy (4)-(6) for all $t \geq 1$.¤

<u>Proposition 2</u>: Any proposition 1 equilibrium is Pareto optimal.

<u>Proof</u>: I will prove that such an equilibrium satisfies the following optimality criterion (see Balasko-Shell, Section 5): there exists $\varepsilon > 0$ such that (the product) $\Pi_1^t[1+r(i)] \geq \varepsilon$ for all $t \geq 1$. This suffices because our economy is a special case of that studied by Balasko-Shell.

Consider, then, any product, $\Pi_1^t[1+r(t)]$, $t \geq 1$, and let t_M be the last date among the first t dates at which $M(t) > 0$. By the lower line of (2) we have

$$\text{(7)} \qquad \Pi_1^t[1+r(t)] \geq \Pi_1^{t_M}[1+r(i)].$$

We also have

$$[1+r(t_M-1)]\,[1+r(t_M)] \begin{cases} = p(t_M+1)/p(t_M-1) & \text{if } M(t_M-1) > 0 \\ \geq p(t_M+1)/\underline{p} & \text{if } M(t_M-1) = 0. \end{cases}$$

By induction, this implies that if t_m is the first date among the first t at which $M(t) = 0$, then

$$\text{(8)} \qquad \Pi_1^{t_M}[1+r(i)] \geq \Pi_1^{t_m-1}[1+r(i)] = p(t_m)/p(t) \geq \underline{p}/\overline{p}.$$

Inequalities (7) and (8) imply that $\Pi_1^t[1+r(i)]$ satisfies the Balasko-Shell optimality criterion.¤

Note that the proof of proposition 2 does make use of the perfect foresight aspect of the equilibrium. Optimality cannot be established using only the bounds on the price of money; namely, $\underline{p} \leq p(t) \leq \overline{p}$ for all t. When $M(t) > 0$, such bounds imply only $1+r(t) \geq \underline{p}/\overline{p}$, which allows for convergence of $\Pi_1^t[1+r(i)]$ to zero. The crucial feature of a perfect foresight sequence is that if $r(t)$ is small because $p(t+1)$ is small, then $r(t+1)$ tends not to be so small because $p(t+1)$ is the denominator of $r(t+1)$. It would certainly seem, however, that if we had a notion of "almost" perfect foresight equilibrium, then optimality could also be established for such an equilibrium.

3. Two Examples

The first example is of an economy with many equilibria under the pure fiat scheme, many that are nonoptimal and one that is optimal. By proposition 2, the nonoptimal ones cannot be equilibria under the hybrid scheme. As we will see, the optimal one is an equilibrium under the hybrid scheme if $\underline{p}M$ is small enough. The second example is of an economy with a unique equilibrium under the pure fiat scheme, one which is optimal and is such that money does not have value. This equilibrium is not an equilibrium under the hybrid scheme.

Example 1: $N(t) = 1$, $u_t(c_1,c_2) = \ln c_1 + \ln c_2$, and $(w_t(t),w_t(t+1)) = (w_1,w_2)$, $w_1 > w_2$.

Under the pure fiat scheme with $M = 1$, say, any nonnegative $p(t)$ sequence satisfying $p(t) = w_1p(t+1)/[2p(t+1)+w_2]$ and $p(t) < w_1$ is an equilibrium (see Figure 2). Thus, as is evident from Figure 2, for any $p(1) \varepsilon [0,(w_1-w_2)/2)$, there exists a nonnegative $p(t)$ sequence with $p(t)$ converging to zero that is an equilibrium. It is well known that all such equilibria are nonoptimal. The only other equilibrium under the pure fiat scheme is $p(t) = (w_1-w_2)/2$ for all $t \geq 1$. It is easily verified that this is an equilibrium under the hybrid scheme if $\underline{p}M < (w_1-w_2)/2$, an equilibrium in which, aside from a transfer to the current or initial old, no taxes or transfers are ever levied.

[INSERT FIGURE 2]

Example 2: $N(t) = 1$, $u_t(c_1,c_2) = \ln c_1 + \ln c_2$,

$$(w_t(t),w_t(t+1)) = \begin{cases} (3/4,\ 1/4); & t = 1,\ 3,\ 5,\ \ldots \\ (1/8,\ 7/8); & t = 2,\ 4,\ 6,\ \ldots. \end{cases}$$

The unique equilibrium under the pure fiat scheme is autarkic and given by $1+r(t) = 1/3$ for t odd and $1+r(t) = 7$ for t even. Since $\Pi_1^t[1+r(t)] \geq 1/3$ for all t, this is an optimal equilibrium.

It is easy to see that this autarkic equilibrium is not an equilibrium under the hybrid scheme. In order that it be one, $q(t) = 0$ for all $t \geq 1$ would have to be consistent with utility maximizing choices given the hybrid rates of return. But $q(t) = 0$ implies $1+r(t) \geq 1$ under the hybrid scheme, and in odd periods $q(t) = 0$ is not utility maximizing at any $1+r(t) \geq 1$.

If $M = 1$ and $\underline{p} \leq 1/8$, then one equilibrium under the hybrid scheme is $(M(t),p(t)) = (1,3\underline{p}/(1+4\underline{p}))$ for t odd and $(M(t),p(t)) = (0,\underline{p})$ for t even. To verify this, one checks that $p(t) \geq \underline{p}$, that $3/4 - c_1 = 3\underline{p}/(1+4\underline{p})$ maximizes $\ln c_1 + \ln c_2$ subject to the t-odd version of (1) and $3/4 - c_1 \geq 0$, and that $1/8 - c_1 = 0$ maximizes the same utility function subject to the t-even version of (1) and $1/8 - c_1 \geq 0$. The equilibrium lifetime allocations are displayed in Figure 3.

[INSERT FIGURE 3]

In the first example, a casual observer of the hybrid scheme equilibrium, $p(t) = (w_1-w_2)/2$ for all t, could mistakenly infer that the cash-in price $\underline{p}$ plays no role. After all, no money is ever turned in or sold at that price. In the second example, the cash-in price is an obvious determinant of the hybrid equilibrium allocation.

4. Concluding Remarks

I have shown that in an appropriately bounded economy, there exists at least one perfect foresight competitive equilibrium under the hybrid scheme and that any such equilibrium is Pareto optimal. As I noted above, the second of these results is not true for the pure fiat scheme, a scheme in which government policy consists of nothing more than the provision of a fixed stock of fiat money. But this advantage of the hybrid scheme vis-a-vis the pure fiat scheme does not come for nothing.

First, as we have seen, it may be necessary to levy taxes under the hybrid scheme. This is presumably a disadvantage of the hybrid scheme vis-a-vis the pure fiat scheme. Second, implementation of an optimal hybrid scheme requires knowledge of at least some features of the economy. Thus, for example, the hybrid scheme I describe does not guarantee optimality in an economy in which $W(t)$ grows exponentially. Optimality would be implied by a hybrid scheme in which $\underline{p}$ and $\overline{p}$ also grow exponentially at the same rate as does $W(t)$.

For these and other reasons, this paper does not constitute a plea for adoption of a monetary system like the hybrid scheme. The main contribution is the demonstration that it is possible to analyze a somewhat complicated monetary system in the context of a model that is true to the notion that assets are acquired only in order to accomplish intertemporal trades. Although economists have been debating the virtues of various monetary systems for a long time, most discussions leave unanswered basic questions concerning feasibility and the desirability of a governmental role. The analysis presented above begins to address such questions.

References

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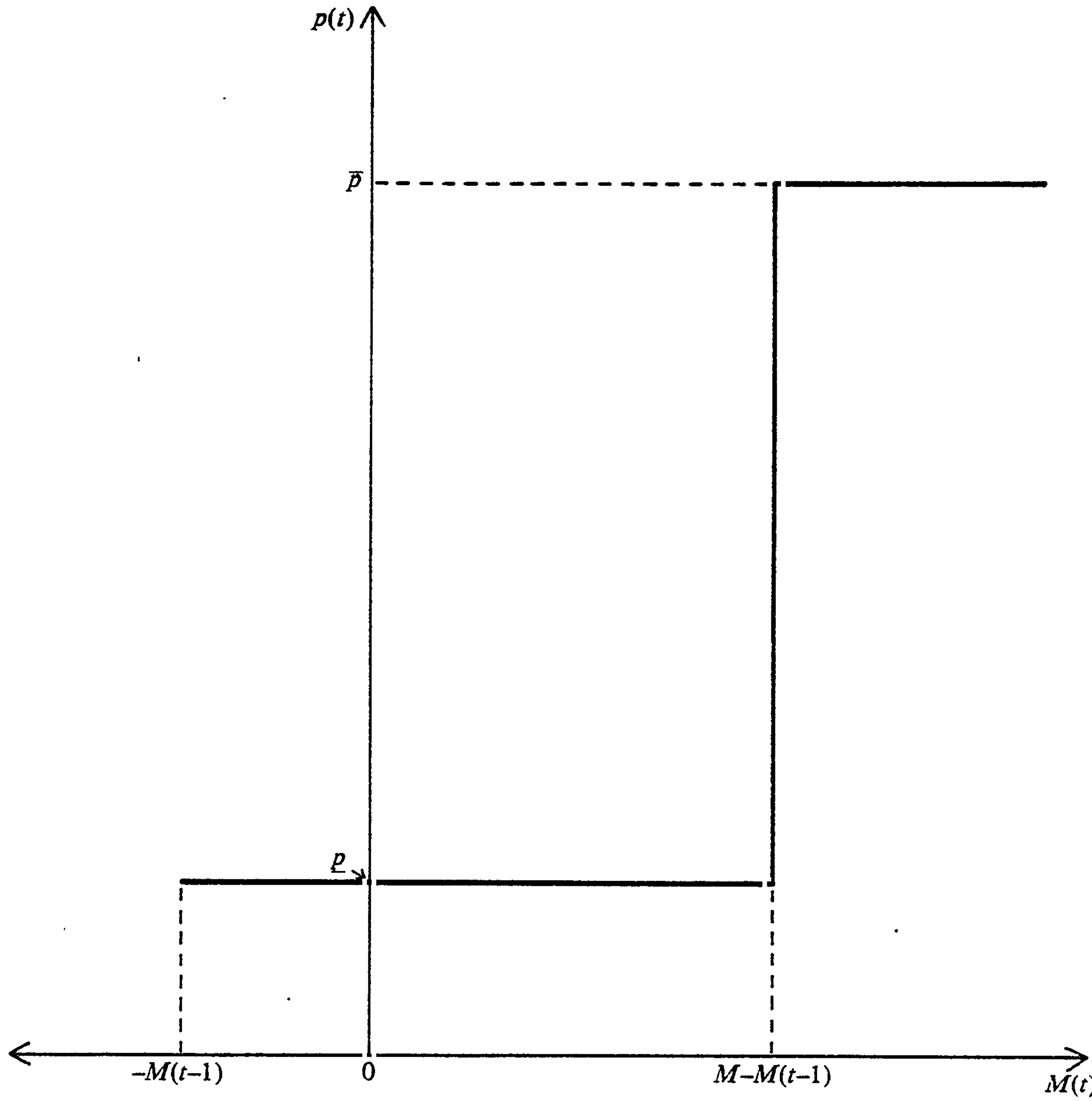

Figure 1 Government supply of money at time t under the hybrid scheme

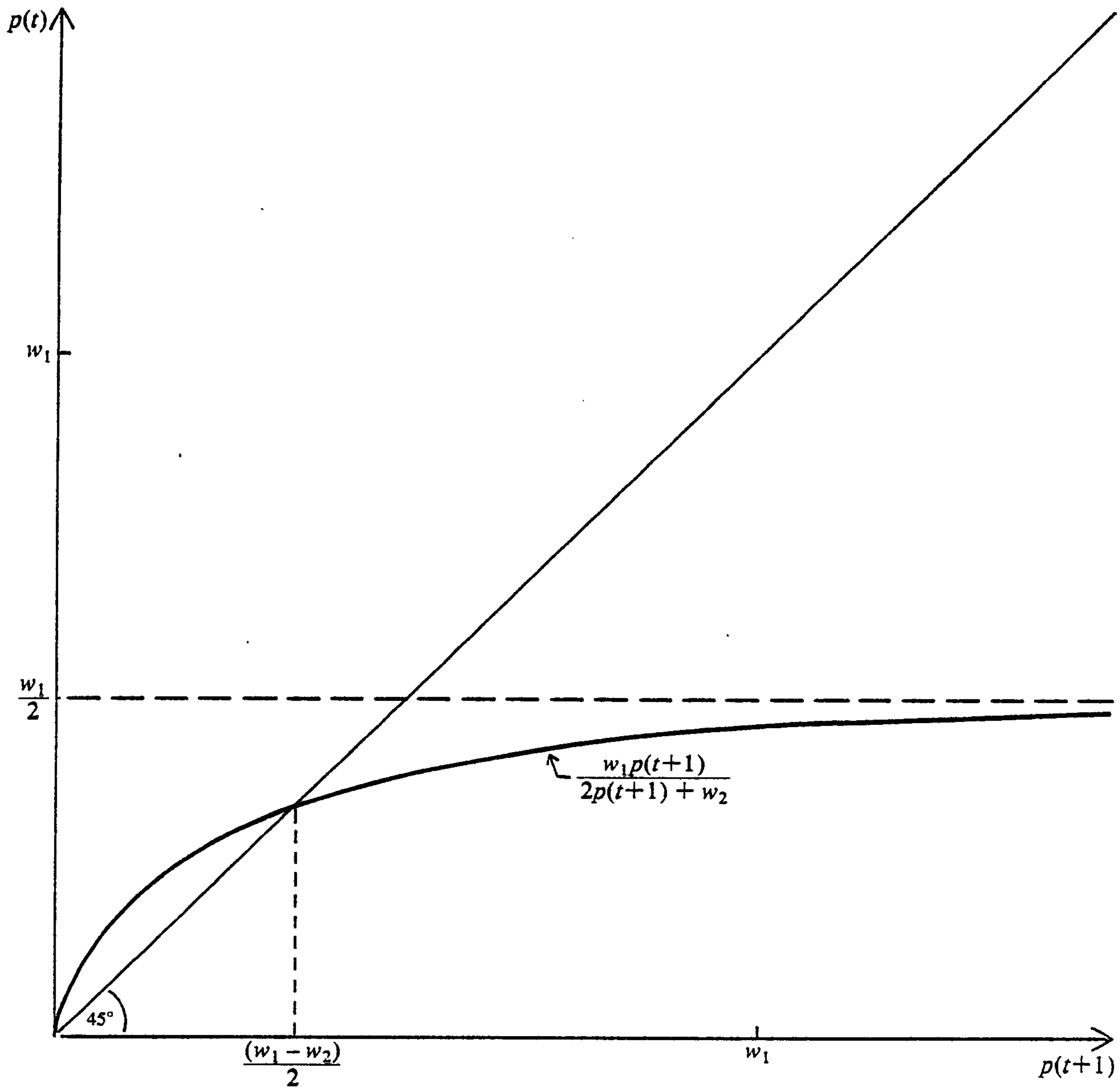

Figure 2 Equilibrium difference equation for example 1 under pure fiat money

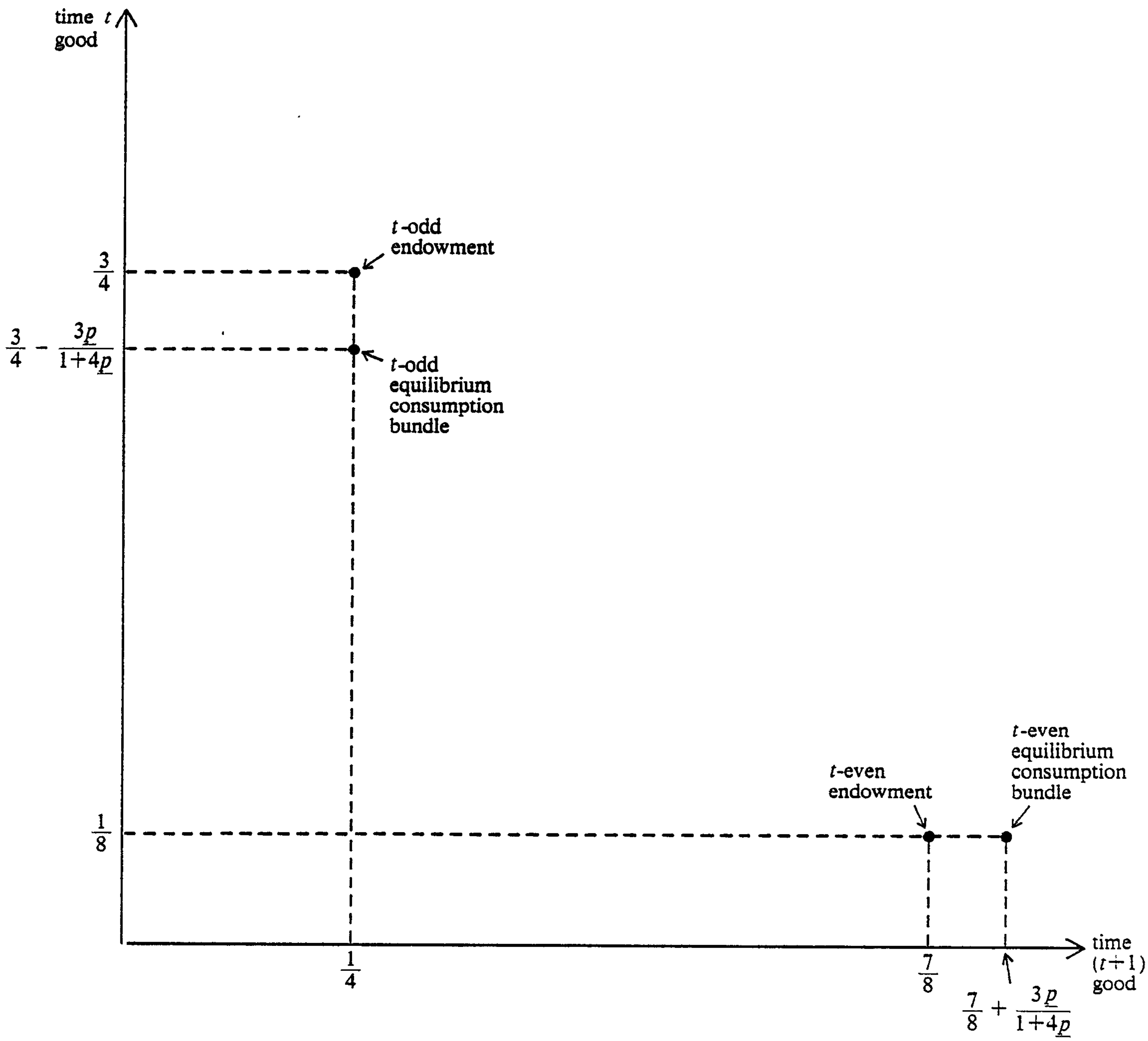

Figure 3 A hybrid equilibrium for example 2