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Do Mergers Lead to Monopoly in the Long Run? Results From the Dominant Firm Model

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ABSTRACT

Will an industry with no antitrust policy converge to monopoly, competition or somewhere in between? We analyze this question using a dynamic dominant firm model with rational agents, endogenous mergers and constant returns to scale production. We find that perfect competition and monopoly are always steady states of this model and that there may be other steady states with a dominant firm and a fringe co-existing. Mergers are likely only when supply is inelastic or demand is elastic, suggesting that the ability of a dominant firm to raise price through monopolization is limited. Additionally, as the discount rate increases, it becomes harder to monopolize the industry, because the dominant firm cannot commit to not raising prices in the future.

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1. Introduction

If there were no antitrust policy limiting mergers, would an industry with constant returns to scale tend to monopoly in the long run? This is a central question in industrial organization. Three key forces have bearing on this issue. First, a monopoly maximizes industry profits, which provides an incentive to consolidate industry capital. Second, as pointed out by Stigler (1950), there is a free-rider problem in the merger process that limits consolidation. Small firms may have an incentive to stay out of a consolidated enterprise, where they can free ride on the attempts of large firms to raise price. Third, small firms have a greater incentive to invest in industry capital than do large firms, because large firms internalize the negative effects of investment on future prices, while small firms do not. This causes the market share of large firms to decline over time in the absence of mergers.[1] Because of these opposing forces, long run industry concentration is unknown when mergers are feasible. The purpose of this paper is to sort out the net effects of these three forces in order to understand how industry concentration will evolve in the absence of regulation.

In order to minimize the number of assumptions, we attempt to answer this question using the simplest possible model that incorporates all three key forces. We develop a dynamic rational expectations model with an endogenous merger process and consider Markov-Perfect Equilibria for simplicity. Our model is based on the Kydland (1979) and Holmes (1996) dynamic dominant firm model of an industry faced with a repeated demand curve for a homogeneous product.[2] The industry is composed of one "large" dominant firm and a continuum of infinitesimally small firms that form a competitive fringe. Every period, there are two stages: a merger stage and an output stage. In the first stage, the dominant firm chooses to acquire capital from the fringe by setting a price at which it will buy or sell as much capital as is supplied. In the second stage, the dominant firm chooses a market price. Competitive fringe firms then pick their production levels for the given market price with the dominant firm supplying the residual demand.

The production technology is constant returns to scale with capital and labor as inputs:

[1] This effect has been widely observed for such classic dominant firms as U.S. Steel, IBM and Kodak. Stigler (1968, p. 108) described this effect as an 'inverted umbrella' held by the dominant firm, that protects and encourages the growth of fringe firms.

[2] See also Gaskins (1971) and Judd and Petersen (1986).

the same for both dominant and fringe firms. We assume that capital is specific to the industry and that current capital is used like seed corn to produce new capital.[3] The production process destroys current capital and outputs new capital in fixed proportion to the consumption good. We assume that capital is required in the production process so that a dominant firm that controlled the entire existing stock of industry-specific capital would be a monopoly and could remain a monopoly forever.

Some words about the relation of this paper to the literature are in order. Several papers examine mergers with game theory without endogenizing the merger process. This literature dates back to Salant, Switzer and Reynolds (1983), who find that Stigler's (1950) free-rider force puts a stark limitation on mergers in a constant marginal costs Cournot world. For instance, in such a world, two firms in an industry of three or more firms would never find it profitable to merge. Similar to our work, Perry and Porter (1985) model a competitive fringe and a dominant firm. Perry and Porter do not model an endogenous merger process, but instead exogenously assume that the fringe will merge into a dominant firm if the total return to the merged entity exceeds what the firms would get separately. An endogenous merger process where the dominant firm could make a joint take-it-or-leave-it offer to the fringe would yield their outcome. With this formulation, any one fringe firm has the power to stop the entire acquisition from occurring. In contrast, we assume that the dominant firm is restricted to setting one price of capital for acquisitions. Our assumption is more appropriate in our context because we treat the fringe firms as a multitude of infinitesimally small players, who cannot individually have the power to affect any acquisition but their own. With our assumption, there is a much stronger free-rider effect against merger, since a dominant firm must pay the fringe its reservation value at the resulting industry structure and not at the original one.[4]

As the above papers did not endogenize the merger process, they cannot be used to examine the long-run effects of allowing for mergers. Recently, there have been several papers

[3] If the capital were not specific to the industry but were rather in perfectly elastic supply, our question would not be an interesting one: perfect competition would certainly prevail since there would be nothing to monopolize.

[4] Our formulation of the merger process is similar to Shleifer and Vishny (1986) who look at the analogous problem of a large shareholder buying shares from a competitive fringe of shareholders.

that have attempted to endogenize the merger process, including Cheong and Judd (1992), Gowrisankaran (1999), and Kamien and Zang (1990). Because these papers have examined oligopoly models, they have had to deal with the complexity of modeling a merger process with many strategic actors. In contrast, by focusing on the dominant firm model, our analysis is vastly simplified, as we need only model the strategic actions of one agent. We feel that three advantages stem from this simplification: First, our model is much more analytically tractable than earlier works such as Gowrisankaran (1999), and hence we can obtain more analytic results. Second, we did not have to make many assumptions about the merger process, in contrast to an oligopoly model where there are many sensible potential processes. Third, with our simple technology, firms in our model merge exclusively for industry profit reasons, which makes our results more easily interpretable. In spite of the simplifications, our model is able to capture the three key forces above, which are intrinsic to an analysis of mergers and long-run concentration.

Using our model, we obtain two immediate results: a monopoly will remain a monopoly forever, and a competitive industry will remain competitive forever. The reason for the monopoly result is that the industry profit force dominates: a monopoly maximizes total industry value and hence cannot gain from a sell-off. The reason for the competition result is that the free-rider force dominates: for a given market structure, a fringe firm maximizes the value per unit of capital, while the dominant firm earns less per unit of capital. In order to acquire capital, a dominant firm must pay the fringe its outside option, which is the value of being a fringe firm given the final industry structure. It follows that an infinitesimally small dominant firm must pay the fringe a higher price than it can earn on each of its new units of capital, and, unlike a larger dominant firm, it does not have an existing base of capital that would benefit from the acquisition. Remaining infinitesimally small is thus the only option that does not yield a negative value.[5]

If the industry starts out in between a monopoly and perfect competition, there are several possibilities for the long-run industry concentration. One possibility is that the indus-

[5] If we had assumed a take-it-or-leave-it merger process of the type discussed earlier, the opposite would be true. The take-it-or-leave-it merger process completely eliminates the free-rider force, and thus any industry would immediately merge to monopoly.

try converges to competition. Another possibility is that the industry converges to monopoly. There can also be other steady states, characterized by some intermediate level of concentration. In these steady states, the dominant firm buys some fringe capital every period, but then invests less than the fringe (the investment force described above) to get back to the same original market share at the start of the next period. We find that the particular steady state that occurs depends on three fundamental model parameters: the elasticities of fringe supply and industry demand and the discount factor.

As fringe supply becomes more elastic, long-run concentration decreases. This result is not surprising because with elastic supply, a fringe with even a small amount of capital can produce a large quantity without raising costs. In the limiting case of perfectly elastic supply, the fringe can earn infinite profits at the monopoly price, and hence a dominant firm would have to pay an infinite price to monopolize the industry.

In contrast, as demand becomes more *inelastic*, long-run concentration decreases. At first glance, this may seem counterintuitive since monopolization raises industry profits the most for inelastic demand. Our result is due to the fact that inelastic demand causes an even larger free-rider problem which dominates the industry profit force and prevents mergers. It is perhaps easiest to see this for the case of merger to monopoly. As demand elasticity approaches 1, a monopoly benefits by cutting back output and raising the price. A hypothetical fringe firm would benefit even more by *expanding* output and still enjoying the higher price. In the limiting case of unit elasticity, prices and hence fringe per-unit profits are infinite but monopoly profits are not. As the dominant firm must pay the fringe its outside option in order to acquire it, in the limit it must pay the fringe an infinite amount but can only earn finite profits. The fact that the dominant firm merges less than when demand is inelastic, combined with the fact that inelastic demand also causes the dominant firm to invest less than the fringe, means that it is only industries with elastic demand that can be monopolized. The surprising conclusion is that there is a natural limitation to the price increase that a dominant firm can sustain in the absence of antitrust policy and that the only industries that tend to monopoly are those industries for which demand is relatively elastic.

The discount factor also plays a key role in affecting long-run concentration. With myopic firms ($\beta = 0$), a dominant firm with market share between 0 and 1 will always acquire a strictly positive amount of capital. The reason for this is that the marginal value of transferring a unit of capital to the dominant firm is more than the fringe value of the capital. To see this, note that the dominant firm can always choose to operate the transferred unit like the fringe and earn the fringe profit on this unit. But, by decreasing output on the transferred unit to factor in the externality on its other units, it can increase overall profits. Because the value of this unit is higher to the dominant firm, it is always optimal to buy some capital. However, since the dominant firm is a monopsony buyer of capital, it may not buy all the capital.

In contrast, with forward-looking firms ($\beta > 0$), the result that there is always a positive merger no longer holds. The above logic breaks down because the dominant firm cannot commit to operating the new unit of capital like a fringe firm in future periods. In fact, the dominant firm chooses to sell off some capital provided its initial share is low and β is large. More generally, an increase in β is conceptually similar to an increase in the elasticity of supply. Thus, just as in the supply elasticity case, we find that mergers are less and less likely as β increases. This is true both near the limit of competition, which goes from being unstable to stable as β increases; near the limit of monopoly, which goes from being stable to unstable; and in between.

The remainder of this paper is divided as follows. In Section 2 we present the model. In Section 3 we present results for the myopic ($\beta = 0$) model. In Section 4, we examine the effect of supply and demand elasticities on the long-run monopolization of the industry. In Section 5, we examine the general ($\beta > 0$) model and discuss how this differs from the myopic case. Section 6 concludes.

2. The Model

We adopt a discrete time model of a dynamic, infinitely lived industry. Firms choose actions to maximize their expected discounted value of profits. Each period, there is a merger process followed by an investment/production process. We model the industry in partial equilibrium with a demand curve that is constant over time. We start by detailing the preferences and technologies and then define the equilibrium of the model.

A. Preferences and Technologies

Demand at price p is $Q = D(p)$. Assume demand is strictly decreasing, and let $p = D^{-1}(Q) \equiv P(Q)$ denote the inverse demand curve. The discount factor is β.

We first explain the technology of the investment/production process (the second stage) and then explain the technology of the merger process (the first stage). At the start of the investment/production process, there is a dominant firm and a competitive fringe, endowed with K_d and K_f units of capital stock, respectively. The capital stock is specific to the industry. The fringe is composed of a continuum of firms, each of whom owns an infinitesimally small amount of K_f and hence is a price taker.

The dominant firm and the fringe firms all have access to the same technology, and they produce homogeneous products. Firms combine industry-specific capital K with non-industry-specific labor L in a production process that produces joint outputs, the consumption good Q and future capital K_{next}. The two outputs are produced in fixed proportions. Let $Q = F(K, L)$ be the production of the consumption good and $K_{next} = \sigma F(K, L)$ be the production of future capital given inputs K and L, where $0 < \sigma < 1$. We assume that current capital K is consumed in the production process. If we interpret Q as a measure of end-of-period capital and define $\delta \equiv 1 - \sigma$, then δ can be interpreted as the depreciation rate, $K_{next} = (1 - \delta)Q$ and σ as the fraction of end-of-period capital that survives into next period.

We make two main assumptions about the production technology. First, we assume that it is constant returns to scale. Thus, the size of the dominant firm does not confer any advantage or disadvantage in the production process relative to fringe firms. Second, we assume that $F(0, L) = 0$. It is therefore impossible to enter this industry without any industry-specific capital. Thus if the dominant firm were to somehow obtain all the industry-specific capital, monopoly forever could be ensured. This sets up the main question of this paper as to whether the industry will ever actually get to this point. In addition to these two main assumptions, we also make the technical assumptions that $F(K, L)$ is strictly concave and is strictly increasing in both arguments (for $K > 0$ and $L > 0$) and that the Inada conditions on L, $\lim_{L\to\infty} F_L(K, L) = 0$ and $\lim_{L\to 0} F_L(K, L) = \infty$, hold for $K > 0$.

In the analysis it will be convenient to utilize a cost function rather than the underlying production function. Let $C(Q, K)$ be the labor cost to produce Q units of the consumption good (and σQ of next-period capital); i.e., $C(Q,K) = \omega L'$ for L', solving $Q = F(K, L')$, given the competitive wage ω. The assumptions on $F(K,L)$ imply that $C(Q,K)$ is homogenous of degree 1, so that $C(Q,K) = KC(Q/K, 1)$. In the analysis we use lower case q to denote output per unit of capital; i.e., $q = Q/K$. Let $c(q) = C(q, 1)$ denote the labor cost per unit of capital necessary to produce q units of output per unit of capital. The assumptions on F imply that c is strictly increasing and strictly convex and that $c'(0) = 0$.

The timing in the investment/production phase follows the textbook treatment of the dominant firm model (see Carlton and Perloff (1994)). The dominant firm first sets an industry price. The fringe firms observe this price and simultaneously decide on production levels. The dominant firm supplies the residual demand at the price. Equivalent to our assumption that the dominant firm chooses price from a residual demand curve, we could assume that it chooses quantity from the same residual demand curve. We will work with this alternate quantity formulation in the following sections as it is notationally more convenient; Perry and Porter (1985) do the same.

We now turn to the merger process, which precedes the investment/production process every period. In this process, the dominant firm posts a price at which it commits to buy or sell all the capital that is supplied or demanded at this price. The fringe firms then simultaneously choose whether or not to sell to the dominant firm given the price and their expectation of the future state. This amount of capital purchased can be zero (corresponding to no merger) or negative (corresponding to a divestiture). It is again more convenient to work with the alternate but equivalent formulation where the dominant firm chooses a quantity of capital to purchase and then picks the price that would yield this quantity.

B. Equilibrium of the Model

We analyze the Markov-Perfect Equilibria of our model in the sense of Maskin and Tirole (1988). This means that we examine equilibria where actions are a function solely of payoff relevant state variables, which are the capital stocks in our case. Let $\left(K_d^\circ, K_f^\circ\right)$ denote the capital stocks held by the dominant firm and fringe before the merger phase has taken place. Let (K_d, K_f) denote the capital stocks *after* the merger phase but *before* the investment/output phase. (Throughout, the superscript "$\circ$" will denote the pre-merger phase, and the absence of a superscript will signify the post-merger phase.) The total capital stock K is the same before and after the merger phase, $K = K_d + K_f = K_d^\circ + K_f^\circ$. It will be convenient to keep track of the state with the total capital stock and the share of the total held by the dominant firm. Let $m^\circ = K_d^\circ / K$ be the industry concentration before the merger phase, and let $m = K_d/K$ denote the concentration after the merger phase.

Define $v_f(m, K)$ to be the discounted value to a fringe firm possessing one unit of capital at the investment/production stage when the aggregate state is (m, K). Analogously, define $v_d(m, K)$ to be the discounted value to the dominant firm *per unit of capital* possessed by the dominant firm. Since the dominant firm holds mK units of capital at this stage, its *total* return is $w_d(m, K) = mKv_d(m, K)$. Throughout the paper, a "v" will denote a return per unit of capital, while a "w" denotes a total return.

The merger phase begins with the dominant firm holding a share m° of the total industry capital. Let $w_d^\circ(m^\circ, K)$ be the maximized total return to the dominant firm at this stage. The dominant firm posts a price of capital p_K at which it commits to purchase the net amount of capital supplied by the fringe at this price (which may be negative). By varying the price p_K, the dominant firm affects the amount of capital supplied, and thus it picks a point on the supply curve of capital. In writing down the dominant firm's problem, it is convenient to have the firm's choice variable be quantity rather than price. In particular, we let the post-merger share m be the choice variable of the dominant firm. Given m, the amount of capital purchased by the dominant firm is $mK - m^\circ K$, total post-merger capital less total pre-merger capital. Given the post-merger share m, the equilibrium price of capital is

$$p_K(m, K) = v_f(m, K).$$

This is the price at which fringe firms are indifferent among buying, selling, or holding onto their capital. Thus the dominant firm makes its merger choice m to solve

$$\begin{aligned} w_d^\circ(m^\circ, K) &= \max_m mKv_d(m, K) - (mK - m^\circ K)p_K(m, K) \qquad (1) \\ &= \max_m mKv_d(m, K) - (m - m^\circ)Kv_f(m, K). \end{aligned}$$

The first term in the objective function is the dominant firm's return entering the investment/production stage with a share of m and thus mK total units of capital. The second term subtracts the amount spent on the acquisition of capital (this subtracts a negative number in the event of a sell-off). Let $\tilde{m}(m^{\circ}, K)$ be the solution to this problem.

Let $v_f^{\circ}(m^{\circ}, K)$ be the return per unit of capital to the fringe before the merger phase. Given the merger policy of the dominant firm, this satisfies

$$v_f^{\circ}(m^{\circ}, K) = v_f(\tilde{m}(m^{\circ}, K), K). \tag{2}$$

Now consider the investment/production phase. Let q_d and q_f denote output per unit of capital for each firm type, so total output in the dominant firm and fringe sectors is $Q_d = mKq_d$ and $Q_f = (1-m)Kq_f$. Recall that when fringe firms make their investment/output decision, the dominant firm has already made its move. Let $\tilde{q}_f(q_d, m, K)$ be the equilibrium output choice (per unit of capital) in the fringe sector, given the choice q_d by the dominant firm and given m and K. This solves the following problem:

$$\begin{aligned} \tilde{q}_f(q_d, m, K) &= \arg\max_{q_f} pq_f - c(q_f) + \beta v_{f,next}^{\circ}(1-\delta)q_f \qquad (3) \\ \text{where} &: \\ p &= P(Q) \\ Q &= mKq_d + (1-m)K\tilde{q}_f(q_d, m, K) \\ v_{f,next}^{\circ} &= v_f^{\circ}(m_{next}^{\circ}, K_{next}) \\ m_{next}^{\circ} &= \frac{mKq_d}{mKq_d + (1-m)K\tilde{q}_f(q_d, m, K)} \\ K_{next} &= (1-\delta)Q. \end{aligned}$$

Because it is infinitesimally small, an individual fringe firm takes the current price p and the future per-unit-of-capital value $v_{f,next}^{\circ}$ as fixed when making its output/investment decision. A choice of q_f yields current revenues of pq_f and a current cost of $c(q_f)$. It also yields $(1-\delta)q_f$ units of capital next period, each unit of which will be worth $v_{f,next}^{\circ}$. The representative fringe takes as given that the other firms in the fringe sector will behave according to $\tilde{q}_f(q_d, m, K)$, so the total fringe sector output will be $Q_f = (1-m)K\tilde{q}_f(q_d, m, K)$. Given this anticipated fringe output and the observed dominant firm output, the representative fringe can calculate what the current price will be and what the future state will be. Note that the dominant firm's next-period pre-merger share m°_{next} is equal to the dominant firm's output share in the current period. This is a convenient feature of the assumption of a fixed-proportions technology for the current consumption good and future capital since next period's total capital equals current output times σ.

The final step is to look at the dominant firm's output/investment decision given that the state is (m, K) in the post-merger phase. It knows that the fringe sector behaves according to $\tilde{q}_f(q_d, m, K)$. As above, it is convenient and equivalent to have the firm's choice variable be quantity rather than price. It chooses q_d to maximize value per unit of initial capital,

$$\begin{aligned} v_d(m,K) &= \max_{q_d} p(q_d)q_d - c(q_d) + \frac{\beta w^\circ(m^\circ_{next}(q_d), K_{next}(q_d))}{mK} \qquad (4) \\ \text{subject to} &: \\ p(q_d) &= P(Q(q_d)) \\ Q(q_d) &= mKq_d + (1-m)K\tilde{q}_f(q_d, m, K) \\ m^\circ_{next}(q_d) &= \frac{mKq_d}{mKq_d + (1-m)K\tilde{q}_f(q_d, m, K)} \\ K_{next}(q_d) &= (1-\delta)Q(q_d). \end{aligned}$$

(Note this is equivalent to maximizing the dominant firm's total value since initial capital is fixed at mK.) Unlike a fringe firm, the dominant firm recognizes that its output/investment choice has an effect on the current and future prices. The dominant firm also factors in the effect of its choice of q_d on the equilibrium fringe supply $\tilde{q}_f(q_d, m, K)$.

Having defined these various functions, we can define a Markov-Perfect Equilibrium (MPE) of this model. In particular, an MPE is a set of functions $\left(v^\circ_f, v_f, v_d, w^\circ_d, \tilde{q}_f, \tilde{q}_d, \tilde{m}\right)$ such that:

(1) The per-unit-of-capital value $v^\circ_f(m^\circ, K)$ solves (2).

(2) The per-unit-of-capital $v_f(m, K)$ is the value to problem (3) for q_d evaluated at $q_d = \tilde{q}_d(m, K)$.

(3) The per-unit-of-capital value $v_d(m, K)$ solves (4).

(4) The total value $w^\circ_d(m^\circ, K)$ solves (1).

(5) The policy function $\tilde{q}_f(q_d, m, K)$ solves (3).

(6) The policy function $\tilde{q}_d(m, K)$ is the solution to (4).

(7) The policy function $\tilde{m}(m^\circ, K)$ solves (1).

We briefly touch on the issue of existence and uniqueness of equilibrium. For the $\beta = 0$ case, a proof is straightforward. It is immediate for this case that there exists a unique fringe response $\tilde{q}_f(q_d, m, K)$ and that this function is continuous. Given this fringe response and given the initial share m°, the dominant firm has a two-stage maximization problem: it first picks m and then q_f. Under straightforward regularity conditions there is a solution to this two-stage maximization problem, and the solution is a continuous function of m°. Things are more complicated for the case of $\beta > 0$. However, in the numerical examples we considered, we did not encounter any difficulties that would suggest that lack of existence or uniqueness of equilibrium is a serious concern.

Now that we have characterized an MPE of this model, we proceed to analyze its properties.

3. Results for the Myopic ($\beta = 0$) Model

This section presents a series of results that show how industry concentration evolves within a period depending upon the initial share m° of capital held by the dominant firm. This section assumes that $\beta = 0$. We begin with the $\beta = 0$ case because it is easier to understand the intuition for this case, and many of the results are the same as in the general case. However, as we will see in Section 5, some of the results are quite different when β is close to 1. To understand why these results are different, it will be useful to first understand the benchmark case where $\beta = 0$.

It is notationally convenient in this section to normalize $K = 1$, as it allows us to eliminate K from the state space. This normalization is without loss of generality, as none of the results of this section depend on K. When $K = 1$, m° and m are the dominant firm's pre-merger and post-merger capital levels as well as shares.

Our first result considers what happens when the industry starts off at the extreme points of monopoly and competition, $m^\circ = 1$ or $m^\circ = 0$. Two general principles emerge here. If we begin the period with monopoly, we end the period with monopoly and forever stay a monopoly. If we begin the period with perfect competition, we end with perfect competition and forever stay competitive. Formally,

PROPOSITION 1. *The equilibrium merger policy function* $\tilde{m}(m^{\circ})$ *satisfies* $\tilde{m}(1) = 1$ *and* $\tilde{m}(0) = 0$.

Proof. Applying (1) to the case of $m^{\circ} = 1$, the dominant firm's problem is

$$\max_{m} \{ m v_d(m) + (1-m) v_f(m) \} . \tag{5}$$

The objective function here is the sum of the dominant firm profit plus the fringe sector profit, which equals total industry profit. Total industry profit is maximized with monopoly, and for $m < 1$ industry profit is strictly less than the monopoly profit. Hence the solution is $m = 1$.

For the case of $m^{\circ} = 0$, applying (1) again, the dominant firm's problem is

$$\max_{m} \{ m v_d(m) - m v_f(m) \} . \tag{6}$$

At $m = 0$, the value in (6) is 0. From (3) we know that a fringe firm maximizes profits per unit of capital, taking industry structure as given, and thus the dominant firm at the same industry structure must earn less profit per unit of capital. Thus, for $m > 0$, $v_d(m) \leq v_f(m)$. Hence $m = 0$ is a solution to the dominant firm's merger choice. Below we show that $v_d(m) < v_f(m)$ for $m > 0$, which implies that $m = 0$ is the unique solution to the merger choice problem. ∎

The monopoly result is due to the industry profits force in our model: a monopoly maximizes industry profits; thus, if the dominant firm were to start at monopoly and sell off capital, industry profits would be smaller and the dominant firm would not even necessarily obtain all of this smaller pie. The competition result is due to the free-rider force of our model: the dominant firm must pay the fringe its reservation price, but since a competitive firm maximizes profits conditional on a given industry structure, the dominant firm has to pay more for capital than it can earn from this capital. Note that Proposition 1 can be generalized to the $\beta > 0$ case with only minor modifications.[6] The two key facts used in the

[6] See Lemma A1 in Gowrisankaran and Holmes (2000).

proof are that a monopoly maximizes industry profits and that a competitive firm maximizes profits taking industry structure as given, and these are both true with $\beta > 0$.

Our next results concern what happens between the extremes of $m^\circ = 0$ and $m^\circ = 1$. These results require an analysis of first-order conditions in the output-investment stage and the merger stage. Under $\beta = 0$, the first-order condition of the fringe firm's problem in the production stage (3) reduces to

$$p - c'(q_f) = 0;$$

i.e., price equals marginal cost. The first-order condition of the dominant firm's problem (4) can be written as

$$MR_d - c'(q_d) = 0, \tag{7}$$

where the dominant firm's marginal revenue is

$$MR_d \equiv p + q_d \frac{dP}{dQ} \left[m + (1-m) \frac{\partial q_f}{\partial q_d} \right]. \tag{8}$$

To interpret (8), consider a decision by the dominant firm to expand its total output by one unit. It gets a price p for the extra unit. But this action will depress the price on the mq_d units that it is already selling. If the dominant firm were a monopoly, then the increase in price would be $\frac{dP}{dQ}$, and the bracketed term in (8) would equal 1. Now suppose that there is a non-zero fringe. Then, any increase in dominant firm quantity is mitigated by a decrease in fringe quantity. Thus, the bracketed term is less than 1 for a dominant firm with a fringe. However, it is straightforward to show that the bracketed term must be strictly positive. (If the bracketed term were not strictly positive, then an increase in dominant firm quantity would not raise total industry output and hence not lower price. But, if price does not fall, then the fringe must not be contracting output. As the dominant firm is expanding output, total industry output is rising, which yields a contradiction.) In combination with the fact that $\frac{dP}{dQ} < 0$, we have shown

LEMMA 1. *A dominant firm with $m > 0$ has $MR_d < p$, and hence $q_d < q_f$.*

Proof. In the appendix, we derive this algebraically in order to obtain an algebraic expression for MR_d, which we use in later algebraic proofs. ■

The difference between a fringe firm and the dominant firm is that a fringe firm's sales are infinitesimally small, and so the second term in (8) drops out. In Figure 1, we illustrate the dominant firm and fringe output decisions. The dominant firm produces at the level q_d, where MR_d equals marginal cost. The profit per unit of capital obtained by the dominant firm is illustrated by the lightly shaded area in the graph between the price line and the marginal cost curve up to q_d. A fringe firm could always choose to produce at q_d and obtain the dominant firm profit. But it can do even better by raising its output to q_f, where price equals marginal cost. The fringe value equals the light gray area (the dominant firm value) plus the dark gray triangle between q_d and q_f. Thus $v_d(m) > v_f(m)$ for $m > 0$. But note that at the extreme where $m = 0$, the dominant firm behaves like a perfectly competitive firm, $q_d = q_f$, and the return per unit of capital is the same, $v_d(0) = v_f(0)$.

Even though the *average* value per unit of capital is higher for the fringe, it can be seen from Figure 1 that the *marginal* value of *transferring* a unit of capital from the fringe to the dominant firm is higher than the fringe value of this capital. Formally,

LEMMA 2.

$$\frac{d\left[mv_d(m)\right]}{dm} = v_d(m) + m\frac{dv_d(m)}{dm} > v_f\left(m\right), \text{ for } m > 0.$$

Proof. The reason for this result is that the dominant firm can commit to not change the price following an acquisition. We derive a formal algebraic proof in the Appendix and go through a graphical proof based on Figure 1 here.

Suppose that a unit of capital that is initially in the fringe sector is transferred to the dominant firm. The dominant firm could always choose to operate the capital unit at the level q_f, the same level the fringe firm would choose. Given that the quantity for the dominant firm's units remains unchanged, the remaining fringe would not change its quantity, and price would remain unchanged. Hence, in this case the dominant firm would earn $pq_f - c(q_f)$ on this new unit of capital, which is the fringe profit $v_f(m)$ (the light gray and dark gray areas in Figure 1). Thus, by setting the same price, the marginal value to the dominant firm of the new capital would be exactly $v_f\left(m\right)$.

Now suppose that the dominant firm were to increase price by decreasing output on this newly transferred capital unit. For each unit decrease in output, dominant firm revenues fall by MR_d and costs fall by marginal cost. But since marginal cost in this region is above MR_d (by Lemma 1), the dominant firm's profit increases as q is decreased. Reducing output from q_f to q_d raises the dominant firm's profit on the newly transferred unit by the black triangle in Figure 1. Thus, the black triangle is the amount by which the marginal benefit of the unit to the dominant firm exceeds the value of the unit to the fringe. Algebraically, we have shown

$$
\begin{aligned}
& \frac{d\left[mv_d(m)\right]}{dm} \\
= \; & [pq_f - c(q_f)] + \int_{q_d}^{q_f} (c'(q) - MR_d)\, dq \\
= \; & v_f(m) + \int_{q_d}^{q_f} (c'(q) - MR_d)\, dq \\
> \; & v_f(m),
\end{aligned}
$$

for $m > 0$. ■

We can use Lemma 2 to understand the dominant firm's incentive to merge. We start by differentiating the dominant firm's acquisition choice (1). Recall that the dominant firm's problem is

$$\tilde{m}(m^\circ) = \arg\max_m mv_d(m) - (m - m^\circ)v_f(m). \tag{9}$$

Define the *merger marginal benefit* to be the slope of this objective function,

$$
\begin{aligned}
& \textit{merger marginal benefit} \\
\equiv \; & v_d(m) + m\frac{dv_d(m)}{dm} - v_f(m) - (m - m^\circ)\frac{dv_f(m)}{dm}.
\end{aligned} \tag{10}
$$

If the dominant firm alters its acquisition decision to purchase one more unit of capital, it can earn a profit of v_d on this new unit (the first term), and acquiring this extra unit drives up the profit on the units of capital the dominant firm is taking into the post-merger phase (the second term). But the dominant firm has to pay v_f for it (the third term), and buying one extra unit raises the price of capital in the capital market (the fourth term).

From Lemma 2, we know that the net effect of the first three terms is strictly positive.

The fourth term limits the incentive of the dominant firm to buy up all the capital because as it does so it raises the price of capital; the dominant firm recognizes its monopsony power in the capital market and factors this effect into its decision. But note that at $m = m^\circ$, this capital market effect is zero since acquisitions are zero, so the fourth term of (10) is zero at this point. Thus a small positive merger is always better than a zero merger. In order to rule out the case of $m < m^\circ$, we impose the following regularity condition:

Assumption 1. The demand function $D(p)$ and the cost function $c(q)$ are such that the equilibrium price $\tilde{p}(m)$ is strictly increasing in m, for $m > 0$.

It is intuitive that this regularity condition should hold, and we can show that it is satisfied for the constant elasticity case considered in Section 4. We then obtain

PROPOSITION 2. *Suppose that $\beta = 0$, that $m^\circ \in (0,1)$, and that Assumption 1 holds. Then, the optimal capital purchase is strictly positive, $\tilde{m}(m^\circ) > m^\circ$.*

Proof. Substituting into (10) from Lemma 2, we can write

$$\begin{aligned} & \textit{merger marginal benefit} \qquad (11) \\ = \ & [pq_f - c(q_f)] + \int_{q_d}^{q_f} (c'(q) - MR_d)\, dq - v_f(m) - (m - m^\circ)\frac{dv_f(m)}{dm} \\ > \ & -(m - m^\circ)\frac{dv_f(m)}{dm}. \end{aligned}$$

Now, $\frac{dv_f}{dm} = q_f \frac{dp}{dm}$ by the envelope theorem. Thus, Assumption 1 implies that $v_f(m)$ is increasing in m. Using this fact, equation (11) shows that *merger marginal benefit* is strictly positive for $m \in [0, m^\circ]$. Thus, the optimal choice of m is strictly greater than m°. ■

While a positive merger is optimal, it is not true that a dominant firm will always merge to monopoly because of its monopsony power in the capital market. Our next result concerns the circumstances under which the dominant firm would merge to complete monopoly. To consider this case, we entertain the following assumption:

Assumption 2. Demand and cost are such that the monopoly price $\widetilde{p}\,(1) < \infty$.

For example, if demand were constant elasticity, this assumption would be satisfied if and only if the elasticity were greater than one. Our result is

PROPOSITION 3. *Suppose Assumptions 1 and 2 both hold. Then, there exists a cutoff* $\hat{m}^{\circ}$ *satisfying* $0 < \hat{m}^{\circ} < 1$ *such that* $\tilde{m}(m^{\circ}) = 1$ *if and only if* $m^{\circ} \geq \hat{m}^{\circ}$. *For* m° *above the cutoff the dominant firm buys the entire stock of fringe capital, and the industry remains a monopoly forever.*

Proof. By Assumption 2, the monopoly price is finite. Thus, the first-order necessary condition (7) holds for $m = 1$, and Lemma 2 can be applied to the $m = 1$ case. Now consider $m^{\circ} < 1$. By Lemma 2, the first three terms of *merger marginal benefit* together are positive when evaluated at $m = 1$. The fourth term is the effect of an additional acquisition on driving up the price of the capital the dominant firm is buying, and this term is negative for positive mergers. However, by continuity, if the dominant firm starts with a market share m° that is close to 1, this term will be close to zero, and hence *merger marginal benefit* will be positive at $m = 1$. As in Proposition 2, Assumption 1 guarantees that immediate merger to monopoly will ensue. By the intermediate value theorem, there exists $\hat{m}^{\circ}$ such that *merger marginal benefit* evaluated at $m^{\circ} = \hat{m}^{\circ}$ and $m = 1$ is exactly 0. As *merger marginal benefit* is monotonically increasing in m°, it follows that for $m^{\circ} < \hat{m}^{\circ}$, immediate merger to monopoly will not occur. ■

Note that the argument in Proposition 3 relies on Assumption 2, that the price is bounded as m goes to 1. If the price were unbounded as monopoly was approached, then the price the dominant firm would have to pay for capital would also be unbounded as monopoly was approached. This issue is discussed further in the next section.

Our next result concerns what happens in the output/investment stage in the event that the initial share m° is less than the cutoff $\hat{m}^{\circ}$ where there is merger to monopoly. For this result, it is useful to use $m^{\circ}_{next}(m)$, which is the next period pre-merger share as a function of the current post-merger share m. Given our assumption of a fixed-proportions technology for output and future capital, the next period pre-merger share equals the current output share,

$$m^{\circ}_{next}(m) = \frac{mq_d}{mq_d + (1-m)q_f} \tag{12}$$

where q_d and q_f are the equilibrium levels of output and are implicitly a function of m. Our result is

Proposition 4. *Suppose that $m^\circ \in (0, \hat{m}^\circ)$, so that there is a positive merger $\tilde{m}(m^\circ) > m^\circ$ but no merger to monopoly, $\tilde{m}(m^\circ) < 1$. Then, the next period pre-merger share is strictly less than the current post-merger share; i.e., $m^\circ_{next}(\tilde{m}(m^\circ)) < \tilde{m}(m^\circ)$.*

Proof. From Lemma 1, we know that $q_d < q_f$; i.e., the dominant firm invests at a lower rate than the fringe. This result is an immediate consequence. ■

Note that if there were no merger phase, this investment force would be the only effect, and the dominant firm's market share would necessarily decline to zero in the long run (see Holmes (1996) and Gaskins (1971)). With mergers, there are two offsetting effects on concentration. The merger phase increases concentration, while the output/investment phase decreases concentration. The net effect is in general ambiguous. Indeed, as we demonstrate in Section 4, there are examples where the two effects cancel each other out, and the industry steady state is characterized by a dominant firm and a fringe.

Our last result concerns the case where the pre-merger share m° is small, in which case there is no ambiguity. Let us first define some notation. Let $F(m^\circ) \equiv m^\circ_{next}(\widetilde{m}(m^\circ))$ be the initial state next period as a function of this period's initial state. Let $s \equiv \frac{dF}{dm^\circ}(0)$ be the slope of next period's concentration as a function of this period's concentration evaluated at $m^\circ = 0$. Then, we can show

Lemma 3.

$$s \equiv \frac{dF}{dm^\circ}(0) = \frac{d\tilde{m}}{dm^\circ}(0). \tag{13}$$

Proof. By Proposition 1, we know that $\widetilde{m}(0) = 0$ and that $q_d(0) = q_f(0)$. Thus, we obtain

$$\begin{aligned}\frac{dF}{dm^\circ}(0) &= \frac{d}{dm^\circ}\left[\frac{\widetilde{m}(m^\circ)\, q_d(\widetilde{m}(m^\circ))}{\widetilde{m}(m^\circ)\, q_d(\widetilde{m}(m^\circ)) + (1-\widetilde{m}(m^\circ))q_f(\widetilde{m}(m^\circ))}\right]\Bigg|_{m^\circ=0} \\ &= \frac{q_f(0)\, q_d(0)}{q_f^2(0)}\frac{d\tilde{m}}{dm^\circ}(0) = \frac{d\tilde{m}}{dm^\circ}(0),\end{aligned}$$

since $\left.\frac{dq_d}{dm}\right|_{m=0}$ and $\left.\frac{dq_f}{dm}\right|_{m=0}$ are finite. ■

The reason for this result is that when the dominant firm market share is zero, the dominant firm and fringe investment rates are equal, since the dominant firm will forever have market share zero by Proposition 1. While the investment rates have a first-order change at $m = 0$, the change in concentration that occurs during the output stage is very small because the initial concentration m° is very small. Thus, next period's concentration m°_{next} will be the same as the end-of-period concentration $\widetilde{m}$ to a first order. Note that this proof is not dependent on $\beta = 0$, provided that Proposition 1 holds and that the derivatives are bounded, both of which we show in Gowrisankaran and Holmes (2000).

Using Lemma 3, we can obtain an unambiguous result about the evolution of industry concentration when m° is small.

PROPOSITION 5. *Suppose that m° is close to zero. The optimal merger and next period's initial concentration are approximately $F(m^\circ) = \tilde{m}(m^\circ) = \frac{4}{3}m^\circ$ to a first-order approximation. Thus, for m° close to zero, the next period pre-merger concentration is strictly greater than the current pre-merger concentration; i.e., $F(m^\circ) > m^\circ$.*

Proof. In Lemma 5 in Section 5 we show that $s = 4/3$ for $\beta = 0$, which gives us the desired result. ■

In Proposition 1, we found that perfect competition is a steady state. Proposition 5 tells us that this steady state is not stable. If the industry starts off with low concentration, it will move towards a higher concentration level in the next period. In contrast, Proposition 3 tells us that the monopoly steady state is stable under Assumption 2. We will see in Section 5 that these results about instability of perfect competition and stability of monopoly are heavily dependent on the assumption of this section that $\beta = 0$.

4. Elasticity of Supply and Demand

This section explores how the evolution of industry concentration depends upon the elasticity of fringe supply and the elasticity of industry demand. We focus on the $\beta = 0$ case in this section for two reasons. First, the results are cleaner and simpler to understand. Second, the results of this section appear to be robust across different values of β.

We further simplify the analysis by focusing on the case of constant elasticity,

$$\begin{aligned} D(p) &= p^{-\varepsilon_D} \\ c(q) &= q^{1+\frac{1}{\varepsilon_S}}. \end{aligned}$$

The parameter ε_D is the elasticity of demand, while the parameter ε_S is the elasticity of fringe supply or, equivalently, the elasticity of marginal cost. Note that we could have allowed for an additional multiplicative parameter for each of these functions, but since the evolution of market share is independent of these multiplicative parameters, they are normalized to one.

Since we will be looking at the dynamics across periods in this section, in principle we have to keep track of that fact that K will be different across periods. However, the constant elasticity model has the convenient property that the evolution of industry concentration is independent of K. Formally,

LEMMA 4. *If demand and marginal costs have the constant elasticity specification, there exists a unique equilibrium. In the equilibrium, the industry transition functions are independent of the capital stock; i.e.,* $\tilde{m}(m^\circ, K) = \tilde{m}(m^\circ, K')$ *and* $m^\circ_{next}(m, K) = m^\circ_{next}(m, K')$, $\forall K, K'$.

Proof. See the Appendix. ■

As a result of Lemma 4, our comparative dynamics analysis is vastly simplified. For a given industry, we can completely characterize industry evolution as a function of one state variable, the industry concentration m or m°. Thus, we rewrite our optimal policy functions as $\widetilde{m}(m^\circ)$ and $m^\circ_{next}(m)$. Moreover, since δ does not affect behavior with $\beta = 0$, δ will affect only future aggregate capital stock levels, and not future industry concentrations. Since the industry transition share functions are independent of the future aggregate capital stock levels, industry evolution and steady state concentration levels will not depend on δ. Hence, for the constant elasticity myopic case, our comparative dynamic results are a function solely of the demand and supply elasticities, ε_D and ε_S.

We begin our comparative dynamics by analyzing the merger policy function $\widetilde{m}(m^\circ)$. In Figure 2a, we have plotted $\widetilde{m}(m^\circ)$ for several values of ε_D for ε_S fixed at 1. In Figure 3a, we plot $\widetilde{m}(m^\circ)$ for several values of ε_S for ε_D fixed at 2; we chose $\varepsilon_D = 2$ instead of 1 because monopoly prices are bounded for these cases. We also plot a dashed 45° line on all the figures. These examples are sufficient to understand some of the general properties that emerge:

REMARK 1. *For a fixed* ε_S, $\widetilde{m}(m^\circ)$ *is increasing in* ε_D. *For a fixed* ε_D, $\widetilde{m}(m^\circ)$ *is decreasing in* ε_S.

Let us start with the result on elasticity of demand, which is illustrated in Figure 2a. At first glance this result is quite counterintuitive. Standard microeconomic theory tells us that inelastic demand is good for a monopolist: it is only with inelastic demand that a monopolist is able to raise price. Thus, the industry profit force should cause more mergers as demand becomes more inelastic. Yet, we find that mergers to monopoly are more and more possible for *elastic* demand. The reason for our result is that the free-rider force prevents the dominant firm from merging as demand becomes more inelastic. Recall that in order to acquire capital, the dominant firm must pay a per-unit price of $v_f(m)$, the fringe reservation price at the resulting market structure. As demand becomes more inelastic, a dominant firm at a given market structure is able to benefit at the output stage by holding back production, which increases market price and hence $v_d(m)$. However, the fringe benefits even more from the increased market price, because it does not have to hold back its production. As $v_f(m)$ increases at an even faster rate than $v_d(m)$, the incentive to merge decreases as demand becomes more inelastic.

It is perhaps easiest to see this result for the case of merger to monopoly. In order to merge to monopoly, a dominant firm with a non-zero fringe would have to pay the fringe a per-unit value of $v_f(1)$, the value from being an infinitesimally small competitive firm in a monopoly environment. Standard monopoly theory tells us that monopoly prices become unboundedly large as the elasticity of demand approaches 1 from above but that monopoly profits $v_d(1)$ remain bounded because the dominant firm is holding back output. Since the monopoly prices become unboundedly large, a hypothetical infinitesimally small fringe firm existing in this environment would also see its profits grow unboundedly large. To summarize, as $\varepsilon_D \to 1$ from above, $v_f(1) \to \infty$, but $v_d(1)$ remains bounded. Thus, the pre-merger market share m° that a dominant firm would need to merge to monopoly approaches 1 as $\varepsilon_D \to 1$.

The same intuition applies to the result on elasticity of supply, which is illustrated in Figure 3a. As supply becomes more elastic, both dominant firm and fringe obtain higher values, but the fringe value increases relatively faster, which makes mergers less and less profitable. At the limiting case of perfectly elastic supply, there are constant marginal costs, and merger to monopoly is impossible since monopolists earn finite profits, while competitive firms earn infinite profits.

We have now explained how demand and supply elasticities affect the amount of capital that the dominant firm chooses to acquire. To understand how industry concentration evolves as a function of these elasticities, it is also necessary to understand the output stage. In Figure 2b, we plot $m^{\circ}_{next}(m)$ for the same values of ε_D as in Figure 2a, and in Figure 3b, we plot $m^{\circ}_{next}(m)$ for the same values of ε_S as in Figure 3a. We argued above that as demand becomes inelastic or supply becomes elastic, the fringe expands investment more and more relative to the dominant firm, which leads to its higher value. Hence, we obtain

Remark 2. *For a fixed ε_S, $m^{\circ}_{next}(m)$ is increasing in ε_D. For a fixed ε_D, $m^{\circ}_{next}(m)$ is decreasing in ε_S.*

From the $\varepsilon_D = 100$ and $\varepsilon_S = 0.01$ cases, we can see that at the limit as demand becomes perfectly elastic or supply becomes perfectly inelastic, a monopoly acts exactly the same as a competitive firm. Thus, at the limit, $m^{\circ}_{next}(m) = m$.

We can combine both remarks in order to understand how long-run industry concentration evolves as a function of the elasticity parameters. To do this, we again use $F(m^{\circ}) \equiv m^{\circ}_{next}(\widetilde{m}(m^{\circ}))$ as the initial concentration next period as a function of this period's initial concentration. We know from Remarks 1 and 2 that as demand becomes more elastic or supply becomes more inelastic, concentration increases at both stages. Given the fact that $m^{\circ}_{next}(m)$ is increasing in m, we obtain

Remark 3. *For a fixed ε_S, $F(m^{\circ})$ is increasing in ε_D. For a fixed ε_D, $F(m^{\circ})$ is decreasing in ε_S.*

In Figure 4, we plot $F(m^{\circ})$ for three values of ε_D (4, 1 and 0) and $\varepsilon_S = 1$ to illustrate several interesting results concerning the evolution of industry concentration.

First, Figure 4 can be used to illustrate and expand on some of the propositions from Section 3. Recall from Proposition 5 that $F(m^{\circ}) \approx 4/3$ for m° close to 0, regardless of the demand or cost structure. This property is evident here, as the three policy function are very similar and have slope close to 4/3 until we reach a dominant firm market share of around $m^{\circ} = .25$. Note also from Figures 2 and 3 that the 4/3 effect occurs entirely at the merger stage, so that $\widetilde{m}(m^{\circ}) \approx 4/3$ for m° close to 0, which is consistent with Lemma 3.

However, closer to monopoly, demand and cost elasticities have huge impacts on the industry transition. For instance, recall from Proposition 4 that if demand is elastic, then merger to monopoly will ensue immediately if m° is sufficiently close to 1. For $\varepsilon_D = 4$, immediate merger to monopoly will occur if $m^{\circ} > .83$. This cutoff increases as the elasticity of demand decreases, and for $\varepsilon_D = 1.1$, the threshold is $m^{\circ} = .995$, which is almost impossible to see on the Figure. For $\varepsilon_D = 0$, immediate mergers to monopoly will never ensue, regardless of how close to monopoly we start out.

Second, Figure 4 can be used to understand how demand and supply elasticities affect long-run industry concentration. Note that an industry steady state will occur for any concentration level m°, where $F(m^{\circ}) = m^{\circ}$, which is graphically equivalent to F being on the 45° line. Furthermore, if and only if F approaches the 45° line from above, the steady state will be stable in the sense that an industry with concentration close to the steady state level will converge to the steady state concentration.

We know that $\widetilde{m}(m^{\circ})$ always lies above the 45° line. For the $\varepsilon_D = 4$ case, $F(m^{\circ})$ also always lies above the 45° line, except at $m^{\circ} = 0$ or 1. The implication is that with $\varepsilon_D = 4$, the industry has exactly two steady states, monopoly and competition, and only monopoly is stable. Thus, with $\varepsilon_D = 4$, an industry with even a small dominant firm will converge to monopoly in finite time. Now consider an industry with $\varepsilon_D = 1.1$. This industry has four steady states, with m^0 of 0, .64, .995 and 1. Moreover, two of these steady states, .64 and 1, are stable. Thus, an industry with $\varepsilon_D = 1.1$ with a dominant firm that has a positive market share but controls less than 99.5 percent of the industry will eventually converge to having a dominant firm that controls 64 percent of the industry. In this oligopolist steady state, the dominant firm starts every period with a 64 percent market share, then purchases some fringe and increases its market share, but then invests at a lower rate than the fringe, and thus falls back to a 64 percent market share at the start of the next period. Similar to the $\varepsilon_D = 1.1$ case, an industry with $\varepsilon_D = 0$ has a stable steady state near $m^{\circ} = .23$. However, this industry has only three steady states (0, .23 and 1), and only the oligopoly one is stable. Thus, for any starting state where a dominant firm and competitive fringe coexist, the industry will converge to an oligopoly with a 23 percent market share.

Lastly, note that, as we can see from the $\varepsilon_D = 4$ case, for very elastic demand,

$F(m^{\circ}) \approx 4/3$ everywhere and not just close to $m^{\circ} = 0$. This implies that mergers to monopoly will occur only when the market share m° is higher than .75. In combination with Remark 1, this shows that an industry with market share of less than 75 percent will not immediately be monopolized.

To further understand the effects of mergers on long-run concentration, Table 1 lists the long-run industry concentration for an industry that starts with a dominant firm with market share $m^{\circ} = .5$. For these results, we have examined 5 values of the demand elasticity ε_D (.5, 1, 1.2, 1.5 and 2) and also 5 values of elasticity of fringe supply ε_S (2, 1, 2/3, 1/2 and 2/5). We find that for inelastic demand and elastic supply, the industry steady state will have both a fringe and dominant firm, as above. As we would expect, as supply becomes more inelastic, the dominant firm is increasingly able to increase its equilibrium market share in the oligopoly equilibrium.

5. The Discount Factor

This section considers the case where firms are forward looking, $\beta > 0$. Some of the general results hold for this case. In particular, the results on elasticities still appear to hold, and competition and monopoly are still steady states of the industry. However, the results on positive mergers (Propositions 2, 3 and 5) break down.

To understand the role that forward-looking behavior plays, it is perhaps instructive to reconsider our proof of Proposition 2, which shows that if $\beta = 0$, a dominant firm with $m^{\circ} \in (0, 1)$ acquires a strictly positive amount. This result does not hold for $\beta > 0$, even in a simple two-period game. From our discussion of the result in Section 3, we know that with $\beta = 0$ the dominant firm can always break even to a first-order following a tiny acquisition by leaving quantity on the new unit unchanged; lowering quantity on this unit yields a first-order gain. When $\beta > 0$, a fringe firm observes the tiny acquisition and knows that even if the dominant firm keeps price the same in the current period, the increased concentration may cause it to want to raise price in future periods. Given this fact, the fringe may want to expand its output in the current period, even for the same price. Thus, the dominant firm may suffer a first-order loss from a tiny acquisition. The underlying difference is that the dominant firm can no longer commit to keeping its behavior fixed following an acquisition, because it cannot commit to its sequence of future prices. Instead, the dominant firm uses divestiture as a substitute for commitment.

The lack of commitment caused by forward-looking behavior reduces the incentive to merge in a fundamental way. This occurs both near the limits of competition and for higher concentration levels. Near the limit of monopoly, the commitment problem goes away because the fringe market share is so small as to make its reactions irrelevant to a first order.[7] However, even near monopoly we observe smaller acquisitions for high β. The logic here is that a high β is very similar to an elastic supply because with a high β, the present value of the quantity produced from a given level of investment increases. Thus, just as in Remark 1, we would expect that a higher β would cause the fringe value to increase relatively more than the dominant firm value, which would cause the dominant firm to acquire less.

In the remainder of the section, we first present some theoretical results near the limits of competition and monopoly. Then we numerically examine the effect of forward-looking behavior in between the limits using the simple case of perfectly inelastic demand.

A. Results Near Monopoly and Competition

We first study the case of low initial concentration m°. Recall from Lemma 3 that s is the slope of both the post-merger and next-period industry concentration evaluated at $m^{\circ} = 0$; i.e., $s \equiv \frac{dF}{dm^{\circ}}(0) = \frac{d\widetilde{m}}{dm^{\circ}}(0)$. For the general β case, we can show

LEMMA 5. *Restrict attention to the set of equilibria that are limits of the equilibria of the finite game as the number of periods becomes infinite. Now, suppose K is at the stationary capital level of perfect competition, $K = K^*_{com}$. Then s is the unique solution to*

$$s = \frac{4}{3} - \left(\frac{4}{3} - \frac{1}{\psi + 1}\right)\beta s^3 \tag{14}$$

for ψ defined by

$$\psi \equiv -\left(\frac{P'K}{c''} + \frac{\beta\sigma^2 K}{c''}\frac{\partial v_f}{\partial K}\right),$$

where all variables are evaluated at the limit of the stationary competitive equilibrium where $m^{\circ} = 0$.

[7] This is the same logic as in Lemma 3, but here it is applied to the fringe instead of the dominant firm.

Proof. This result follows from tedious calculations. See Gowrisankaran and Holmes (2000). ■

Note that for $\beta = 0$, (14) reduces to $s = \frac{4}{3}$, which is what was claimed in Proposition 5 for the $\beta = 0$ case. The key point there was that the slope was strictly *greater* than one, which means that for small m° the dominant firm makes strictly positive acquisitions of capital. Our result here is that for high β the slope s is strictly *less* than one, which means that for small m°, the dominant firm is selling off some capital. Since concentration also decreases during the production stage, concentration necessarily declines over the two stages. Formally,

PROPOSITION 6. *Suppose that as β is varied, K is adjusted to equal the stationary capital level of perfect competition; i.e., $K = K^*_{com}(\beta)$. There exists $\hat{\beta} < 1$, such that if $\beta > \hat{\beta}$, then the slope $s(\beta, K^*(\beta)) < 1$. For such β, if m° is close to zero, the dominant firm sells off an amount of capital in the period approximately equal to a fraction $1 - s$ of its initial stock. Thus, for such β, the competitive steady state is stable.*

Proof. We show in the appendix that $\lim_{\beta \to 1} \psi > 0$. The result then follows from standard algebra. ■

For the case of constant elasticity of demand and supply it is straightforward to numerically calculate the slope s. For fixed ε_D and ε_S, the slope is monotonically decreasing in β, and so there is a cutoff $\hat{\beta}(\varepsilon_D, \varepsilon_S)$ such that competition is stable if and only if $\beta > \hat{\beta}(\varepsilon_D, \varepsilon_S)$. Table 2 shows the cutoff for various levels of ε_D and ε_S (the cutoff is independent of δ).

The key thing to note about Table 2 is that it tells the same story as Remark 1 regarding the effects of changing ε_D and ε_S. As the elasticity of demand increases or the elasticity of supply decreases, the equilibrium incentive to merge increases.[8] In particular, the cutoff $\hat{\beta}$ is clearly increasing in ε_D and decreasing in ε_S.

We next consider the case of m° close to 1. We find that if β is sufficiently high, the monopoly outcome is unstable.

[8] This result can be generalized to include all downward sloping demand curves, not just constant elasticity demand curves.

Proposition 7. *Given a demand curve $Q(\cdot)$, cost structure $c(\cdot)$, and discount factor δ, there exists a $\overline{\beta} < 1$ such that if $\beta > \overline{\beta}$, there does not exist an equilibrium where the steady state monopoly outcome is stable.*

Proof. The proof rests on the fact that there exists $\overline{\beta} < 1$ such that for $\beta > \overline{\beta}$, the fringe value given a monopoly is infinite, $v_f(1, K) = \infty$; we prove this formally in the appendix. To see this, note that in the monopoly steady state, the monopoly output level just offsets depreciation. As the fringe investment rate is higher than the monopolist's, it is above that needed to offset depreciation. Thus, a hypothetical fringe starting with one unit of capital grows unboundedly large in the long run. If β is large, these future unboundedly high output levels are weighted heavily, and the fringe earns more and more in present value with each subsequent period. If there did exist a stable monopoly steady state, then for a given K, for m° close enough to 1, the concentration and price would converge to this steady state. But fringe values would then be infinite for such m°, which is inconsistent with equilibrium. ■

We can again calculate cutoffs for the case of constant elasticity of demand and supply. To do this, note first that the monopoly price satisfies

$$p^*_{mon} = \frac{\varepsilon_D}{\varepsilon_D - 1}\left[(1-\beta)c'(q^*) + \beta(1-\delta)c(q^*)\right], \tag{15}$$

where $q^* = \frac{1}{1-\delta}$ is the output per unit of capital that just offsets depreciation (and leads to a constant output in each period). The expression in brackets can be interpreted as dynamic marginal cost. The stationary monopoly price is the standard markup of $\varepsilon_D/(\varepsilon_D - 1)$ over marginal cost.

For a given ε_D and ε_S, we can again evaluate a cutoff $\overline{\beta}$ above which the fringe value is infinite at the monopoly steady state price. For $\beta \leq \overline{\beta}$, we can directly apply Proposition 4 to show that monopoly is stable. To see this, near the limit of $m = 1$, Lemma 2 holds because even though there is a lack of commitment, the small size of the fringe makes this a second order problem. Thus, the proofs of Propositions 2 and 4 go through, and there is merger to monopoly if the initial share is high.

In Table 3, we have calculated the cutoff levels $\overline{\beta}$. Table 3 tells the same story as Remark 1 and Table 2 regarding the effects of changing ε_D and ε_S. Note that there are some parameter values ($\varepsilon_D = 1$, $\varepsilon_S = .1$, $\beta = 0.945$) for which both monopoly and competition are stable.

B. Numerical Results

We know that allowing for forward-looking firms decreases the incentive to merge near the limits of competition and monopoly. We would like to understand whether this result is true in between these limits. In Figure 5, we have plotted the industry transition function $F(m^\circ)$ for the case of $\varepsilon_D = 0$ and $\varepsilon_S = 1$ and for three different values of β, 0, 0.25 and 0.95. We chose the perfectly inelastic case, because it simplifies our analysis. Total quantity must be equal to 1, and hence total capital stock must be $1-\delta$. Thus, just as in Section 4, we again do not need to include K in the state space. We also found that the industry transition functions are not affected by δ.[9]

The results in Figure 5 show that the industry becomes more competitive for high β. In particular, we find that as β decreases, the stable steady state of $m^\circ = .23$ from Figure 4 decreases. For β above the critical cutoff $\hat{\beta}(0,1) = .25$, competition is the unique stable steady state.

6. Concluding Remarks

In this paper, we have developed a dynamic model of endogenous mergers. While our model is simple and relatively tractable, it captures three key forces that influence industry evolution when mergers are allowed: the fact that monopolization allows firms to raise prices, the free-rider effect that limits the ability of firms to merge, and the investment effect that causes the market share of dominant firms to decline in the absence of mergers.

Our model generates several interesting results. We find that competition and monopoly are steady states of the industry and that there may be other steady states with a dominant firm and fringe co-existing. Comparative statics results show that mergers and monopolization are likely to occur in industries with elastic demand and/or inelastic supply. While

[9] Because of convergence difficulties, we computed strategies only for $m^\circ \in [0, 0.7]$. As we found that this set maps onto itself in equilibrium, this restriction does not affect the results.

allowing for firms to be forward-looking does not change these results, it does change other results by introducing a lack of commitment. In particular, when $\beta = 0$, a dominant firm that co-exists with a fringe will always find it optimal to merge a positive amount, a very small dominant firm will increase its market share from period to period, and a dominant firm that is close to monopoly will monopolize the industry provided that the monopoly price is finite. For high values of β, all of these results are reversed.

Our model departs from the recent game theoretic literature on mergers in that we have only one strategic agent, the dominant firm. As we mentioned in Section 1, we feel that modeling one strategic agent allows us to obtain analytic and interpretable results while still capturing the three key forces that influence industry evolution. Because we are capturing these key forces, many of the insights that we have gleaned from this model, such as the results on elasticities and on the effects of forward-looking firms, do not depend fundamentally on the fact that we have one strategic actor. Moreover, because of our result that a competitive firm will never find it optimal to acquire any capital, we expect that allowing for fringe firms to act as strategic buyers in the capital market would not create an additional player with positive size. Thus, our results are likely to be robust to allowing multiple strategic agents.

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Appendix

Lemma 1. *A dominant firm with $m > 0$ has $MR_d < p$, and hence $q_d < q_f$.*

Proof. The equilibrium fringe output level $q_f(q_d, m)$, given q_d and m, solves the fringe first-order necessary condition (FONC)

$$P(mKq_d + (1-m)Kq_f) - c'(q_f) = 0. \tag{16}$$

Differentiating yields

$$P'\left[mK + (1-m)K\frac{\partial q_f}{\partial q_d}\right] - c''(q_f)\frac{\partial q_f}{\partial q_d} = 0.$$

Solving for $\frac{\partial q_f}{\partial q_d}$ yields

$$\frac{\partial q_f}{\partial q_d} = \frac{P'mK}{-(1-m)KP' + c''_f}.$$

Now consider the dominant firm's problem,

$$\max_{q_d} P(mKq_d + (1-m)Kq_f)q_d - c(q_d).$$

The FONC is

$$\begin{aligned} P + P'\left[mK + (1-m)K\frac{\partial q_f}{\partial q_d}\right] q_d - c'_d &= 0 \qquad (17)\\ P + P'\left[mK + (1-m)K\frac{P'mK}{-(1-m)KP' + c''_f}\right] q_d - c'_d &= 0 \\ MR_d - c'_d &= 0 \end{aligned}$$

for

$$MR_d = P + P'\left[\frac{c''_f}{-(1-m)KP' + c''_f}\right] mKq_d. \tag{18}$$

It is immediate that $MR_d < P$. ■

Lemma 2.

$$\frac{d\left[mv_d(m)\right]}{dm} = v_d(m) + m\frac{dv_d(m)}{dm} > v_f(m), \text{ for } m > 0.$$

Proof. Differentiating the fringe FONC (16) with respect to m yields

$$P'\left[K(q_d - q_f) + (1-m)K\frac{\partial q_f}{\partial m}\right] = c_f''\frac{\partial q_f}{\partial m}$$

so

$$\frac{\partial q_f}{\partial m} = \frac{-P'K(q_f - q_d)}{c_f'' - P'(1-m)K}.$$

The dominant firm value is

$$v_d = \max_{q_d} P(mKq_d + (1-m)Kq_f(q_d, m))q_d - c(q_d).$$

By the envelope theorem,

$$\frac{\partial v_d}{\partial m} = P'\left[K\left(q_d - q_f\right) + (1-m)K\frac{\partial q_f}{\partial m}\right] q_d$$

so

$$\begin{aligned}
\frac{\partial v_d}{\partial m} &= P'\left[-K(q_f - q_d) + (1-m)K\frac{-P'K(q_f - q_d)}{c_f'' - P'(1-m)K}\right] q_d \\
&= \frac{-P'K(q_f - q_d)c_f''q_d}{c_f'' - P'(1-m)K}.
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{d\left[mv_d(m)\right]}{dm} &= v_d(m) + m\frac{dv_d(m)}{dm} \\
&= pq_d - c(q_d) + m\frac{-P'K(q_f - q_d)c_f''q_d}{c_f'' - P'(1-m)K} \\
&= v_f(m) + p(q_d - q_f) - (c(q_d) - c(q_f)) + m\frac{-P'K(q_f - q_d)c_f''q_d}{c_f'' - P'(1-m)K} \\
&= v_f(m) + [MR_d q_d - c(q_d)] - [MR_d q_f - c(q_f)] \\
&= v_f(m) + \int_{q_d}^{q_f} \left(c'(q) - MR_d\right) dq,
\end{aligned}$$

where the integral is strictly positive since $q_f > q_d$ for $m > 0$ and $c'(q) > MR_d$ for $q > q_d$. ■

Lemma 4. *If demand and marginal costs have the constant elasticity specification,* *there exists a unique equilibrium. In the equilibrium, the industry transition functions are independent of the capital stock; i.e.,* $\tilde{m}(m^\circ, K) = \tilde{m}(m^\circ, K')$ *and* $m^\circ_{next}(m, K) = m^\circ_{next}(m, K')$, $\forall K, K'$.

Proof. It simplifies the algebra to renormalize the cost function to

$$c(q) = \frac{1}{1+\frac{1}{\varepsilon_S}} q^{1+\frac{1}{\varepsilon_S}}$$

(in the text the multiplicative constant is left out).

For any m, define $\overline{q}_d(m)$ and $\overline{q}_f(m)$ to satisfy

$$\begin{aligned}
(m\overline{q}_d + (1-m)\overline{q}_f)^{-1/\varepsilon_D} - \frac{(m\overline{q}_d + (1-m)\overline{q}_f)^{-1-1/\varepsilon_D}}{(1-m)\varepsilon_S \overline{q}_f^{\frac{\varepsilon_D}{\varepsilon_S}+1} + \varepsilon_D} m\overline{q}_d - \overline{q}_d^{1/\varepsilon_S} &= 0 \\
(m\overline{q}_d + (1-m)\overline{q}_f)^{-1/\varepsilon_D} - \overline{q}_f^{1/\varepsilon_S} &= 0.
\end{aligned} \tag{19}$$

Assuming that $\varepsilon_D > 0$, it can be shown that for each $m \in [0,1]$, there exists a solution to (19) where $\overline{q}_d > 0$ and $\overline{q}_f > 0$. Now define

$$\begin{aligned}
q_d(m, K) &= \overline{q}_d(m) K^{-\omega} \\
q_f(m, K) &= \overline{q}_f(m) K^{-\omega},
\end{aligned} \tag{20}$$

where

$$\omega \equiv \frac{\varepsilon_S}{\varepsilon_D + \varepsilon_S}.$$

Substituting (20) into the dominant firm FONC (17), and the fringe FONC (16), we can factor out the terms involving K resulting in (19). Therefore, at these output levels the FONCs are satisfied. It is immediate that when the fringe FONC is satisfied, the fringe is at the unique global optimum output level. With more tedious arguments we can show that for the constant elasticity case, in the dominant firm problem there is a unique output level solving its FONC, are and this is the global optimum for the firm. Hence given (m, K), q_d and q_f from (20) are the unique equilibrium outputs of the output-investment stage. Since the ratio of the equilibrium values of q_d and q_f is independent of K, it is immediate that

$m^{\circ}_{next}(m, K)$ is independent of K.

It is straightforward use (20) to calculate the value functions $v_f(m, K)$ and $v_d(m, K)$ and to determine that they can be written in the form of

$$\begin{aligned} v_d(m, K) &= K^{-\zeta} f_d(m) \\ v_f(m, K) &= K^{-\zeta} f_f(m) \end{aligned}$$

where

$$\zeta \equiv \frac{\varepsilon_S + 1}{\varepsilon_D + \varepsilon_S}.$$

Thus the objective function in the merger problem is multiplicatively separable in K, so the optimal merger $\tilde{m}(m^{\circ}, K)$ is independent of K. ■

Proposition 6. *Suppose that as β is varied, K is adjusted to equal the stationary capital level of perfect competition; i.e., $K = K^*_{com}(\beta)$. There exists $\hat{\beta} < 1$, such that if $\beta > \hat{\beta}$, then the slope $s(\beta, K^*(\beta)) < 1$. Thus for such β, if m° is close to zero, the dominant firm sells off an amount of capital in the period approximately equal to a fraction $1 - s$ of its initial stock. For such β, the competitive steady state is stable.*

Proof. As discussed in the text, it is sufficient so show that

$$\lim_{\beta \to 1} \psi(\beta) > 0. \tag{21}$$

Note that $q^* = \frac{1}{\sigma}$ in a stationary equilibrium, so that $\sigma q^* = 1$. Thus c'' does not vary with β. It is also straightforward to show that $\frac{\partial v_f}{\partial K} \leq 0$. Thus it is sufficient to show that the limit of $P'K$ is nonzero. It can be shown that the stationary price is

$$p^* = (1 - \beta)c'(q^*) + \beta \sigma c(q^*).$$

Now

$$\lim_{\beta \to 1} p^* = \sigma c(\frac{1}{\sigma}).$$

Because p^* remains bounded and above 0 in the limit,

$$\begin{aligned}\lim_{\beta\to 1}\left(-P'K\right) &= \lim_{\beta\to 1}\left(-P'(Q)\sigma Q\right)\\ &= -\left(\lim_{\beta\to 1}P'(Q)\right)\sigma\lim_{\beta\to 1}Q\\ &> 0\end{aligned}$$

so (21) holds. ∎

Proposition 7. *Given a demand curve $Q(\cdot)$, cost structure $c(\cdot)$, and discount factor δ, there exists a $\overline{\beta} < 1$ such that if $\beta > \overline{\beta}$, there does not exist an equilibrium where the steady state monopoly outcome is stable.*

Proof. We show here that $v_f(1, K) = \infty$ for high enough β. First, note that in a stationary equilibrium, a monopoly produces at a rate $q^*_{mon} = \frac{1}{1-\delta}$. As a monopoly sets a price above marginal cost, $p^*_{mon}(\beta) > c'\left(\frac{1}{1-\delta}\right)$. Moreover, it can be shown that the price will stay bounded above marginal costs as $\beta \to 1$. Furthermore, since marginal costs are increasing, marginal costs are greater than average variable costs, hence $p^*_{mon}(\beta) > c\left(\frac{1}{1-\delta}\right)$.

Now for a given β, consider a hypothetical competitive fringe firm existing at the steady state monopoly environment $p^*_{mon}(\beta)$. Suppose that the fringe invests at a rate $q_f = \frac{1}{\beta(1-\delta)}$ from every period forward; note that $q_f > q^*_{mon}$. This strategy would yield a per-unit profit to the fringe of $p^*_{mon}(\beta) - c\left(\frac{1}{\beta(1-\delta)}\right)$ in the current period. Moreover, the profits next period are $\frac{p^*_{mon}(\beta)-c\left(\frac{1}{\beta(1-\delta)}\right)}{\beta}$, which has the same present value of $p^*_{mon}(\beta) - c\left(\frac{1}{\beta(1-\delta)}\right)$. Repeating this logic, the fringe firm will earn the same present value of profits for every future period.

Thus, provided $p^*_{mon}(\beta) - c\left(\frac{1}{\beta(1-\delta)}\right)$ is positive, the fringe value per unit of capital is infinite. Now, as $p^*_{mon}(\beta)$ is bounded above $c\left(\frac{1}{1-\delta}\right)$, for high enough β, $p^*_{mon}(\beta) > c\left(\frac{1}{\beta(1-\delta)}\right)$ and the fringe value will be infinite. Even if K is not $K^*_{mon}(\beta)$, $v_f(1, K)$ will be infinite for the same set of β, since the monopoly capital stock will eventually converge to $K^*_{mon}(\beta)$, and the fringe can earn constant profits per period in present value starting in some future period when p is sufficiently close to $p^*_{mon}(\beta)$. ∎

Figure 1
Profit Maximizing Quantities
For Dominant Firm and Fringe

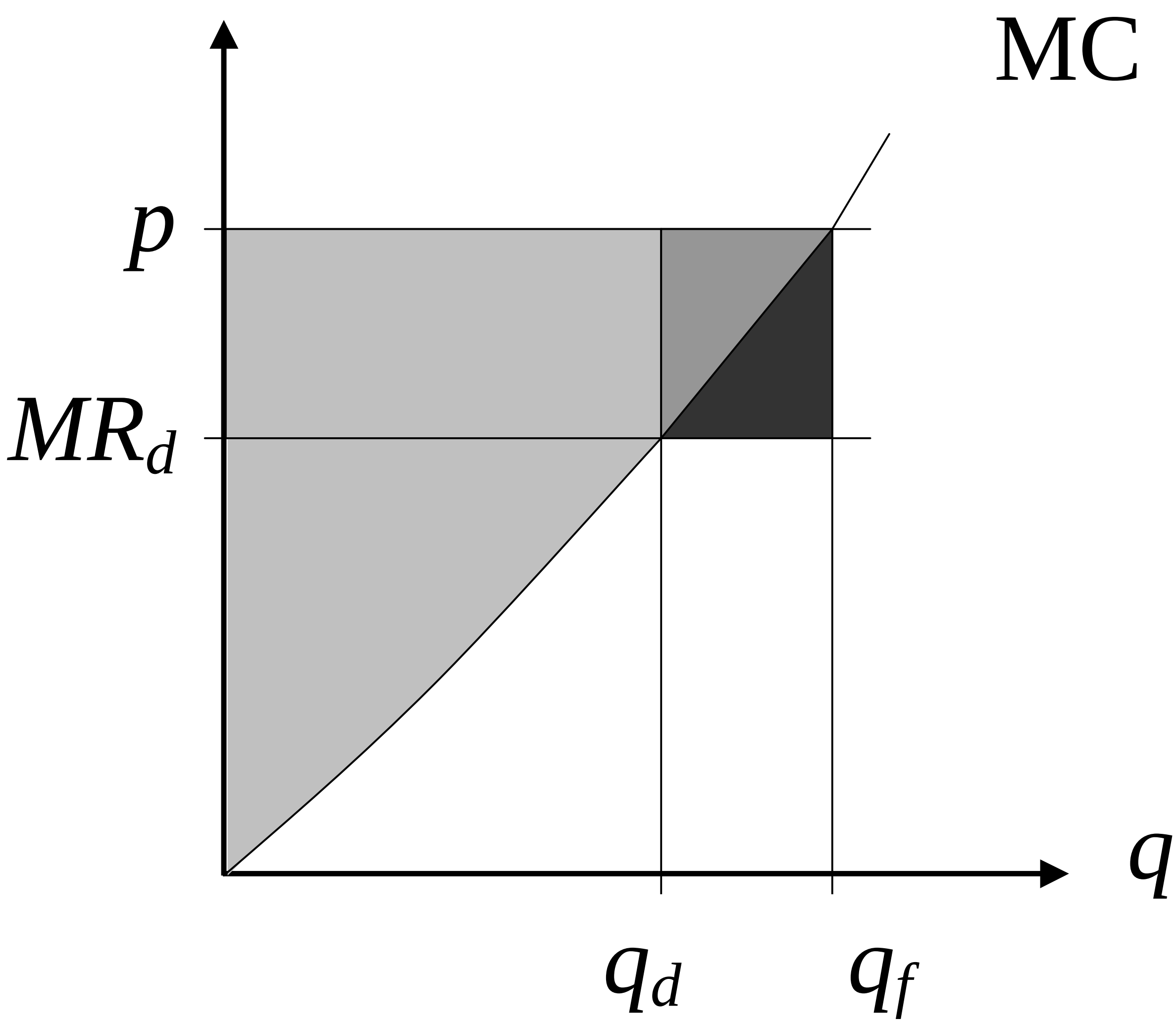
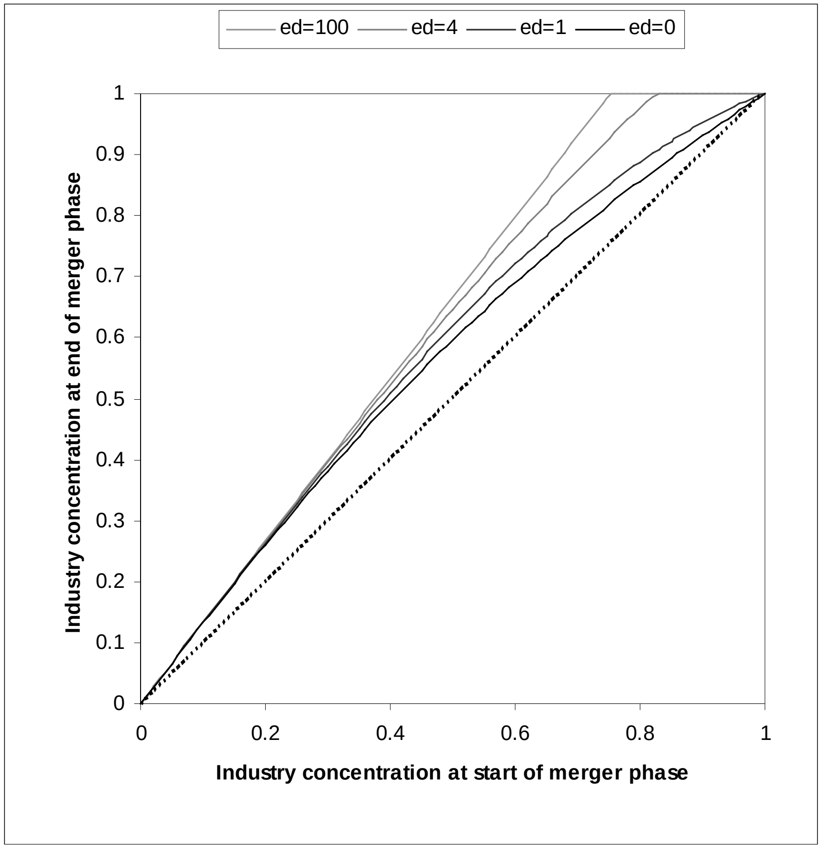


Figure 2: Industry Evolution As Elasticity of Demand (ε_D) Varies; ε_S held fixed at 1

(a) $m(m^{\circ})$



(b) $m^{\circ}_{next}(m)$

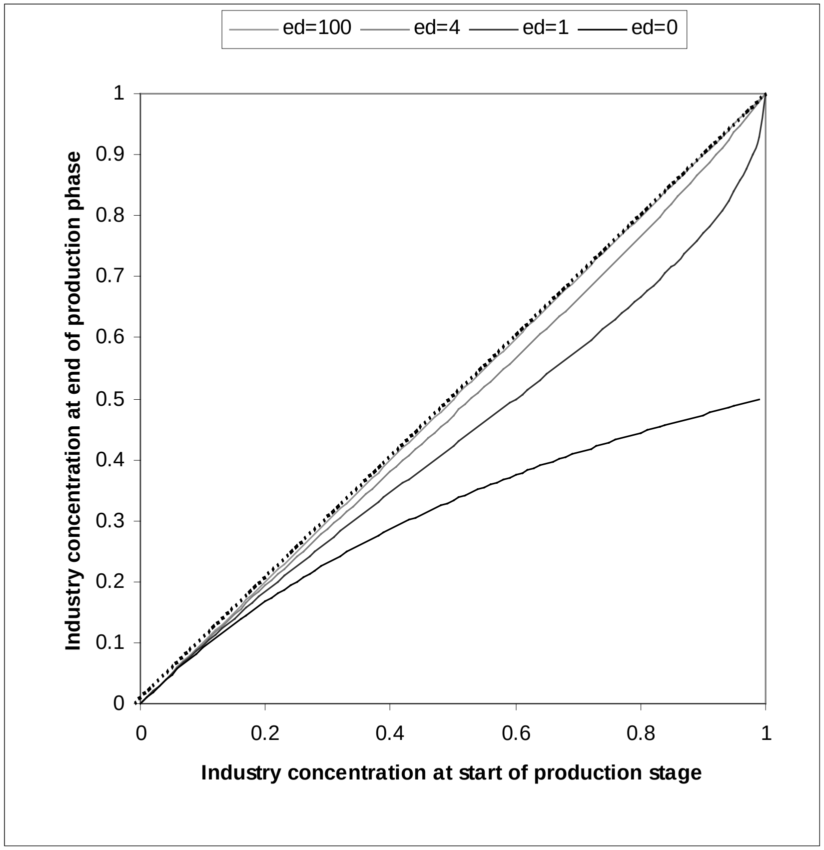
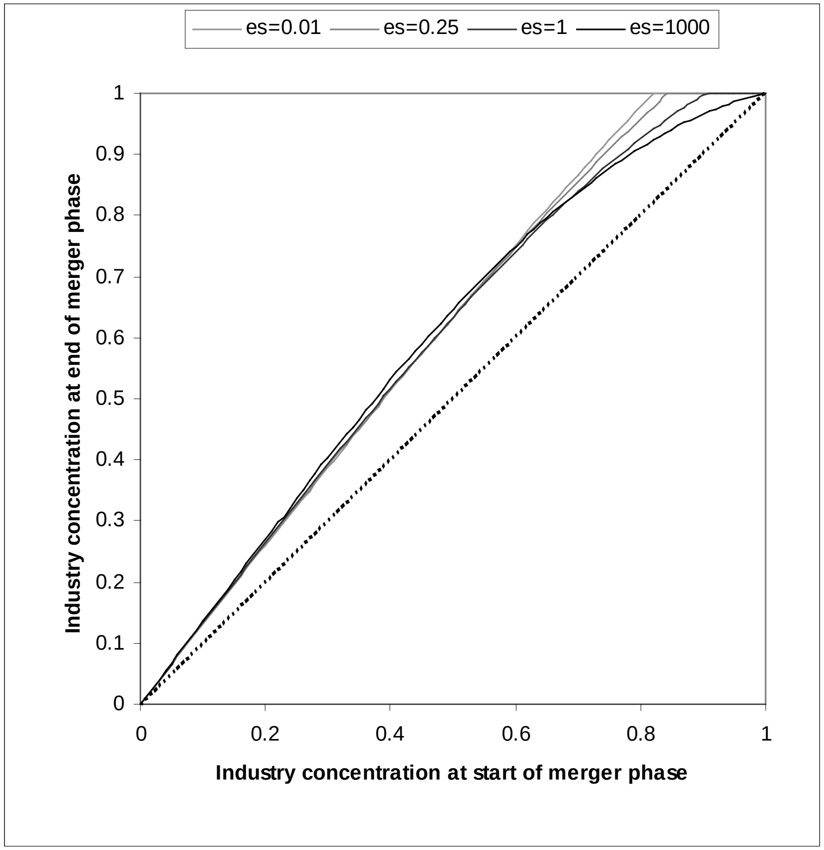


Figure 3: Industry Evolution As Elasticity Of Supply (ε_S) Varies; ε_D held fixed at 2

(a) $m(m^{\circ})$



(b) $m^{\circ}_{next}(m)$

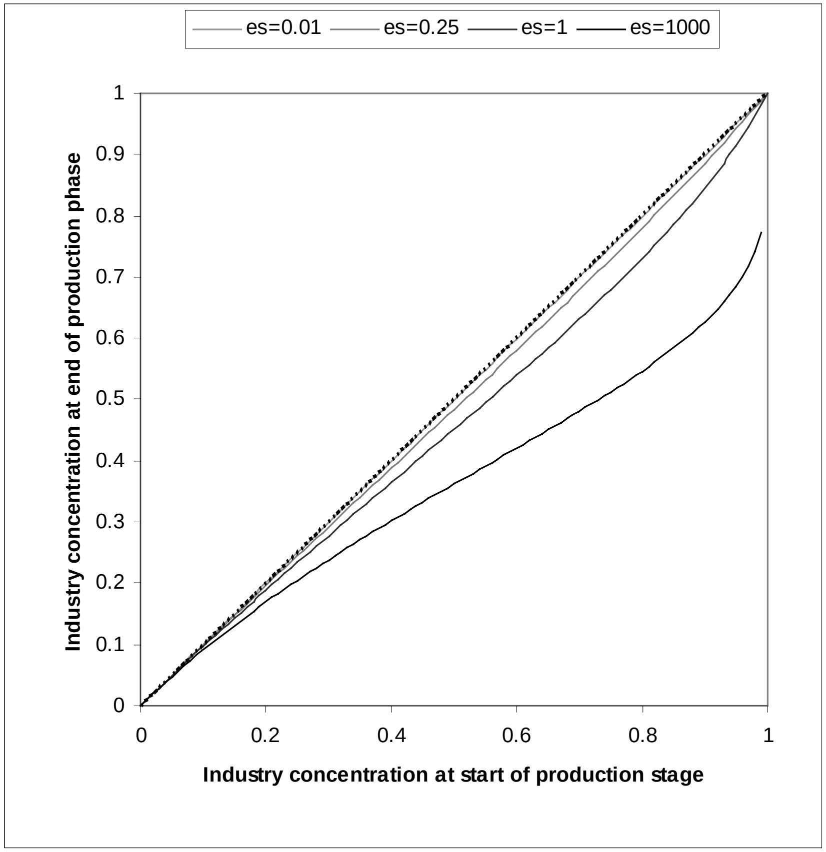


Figure 4: Industry Transition F(m°)
As ε_D Varies; ε_S held fixed at 1

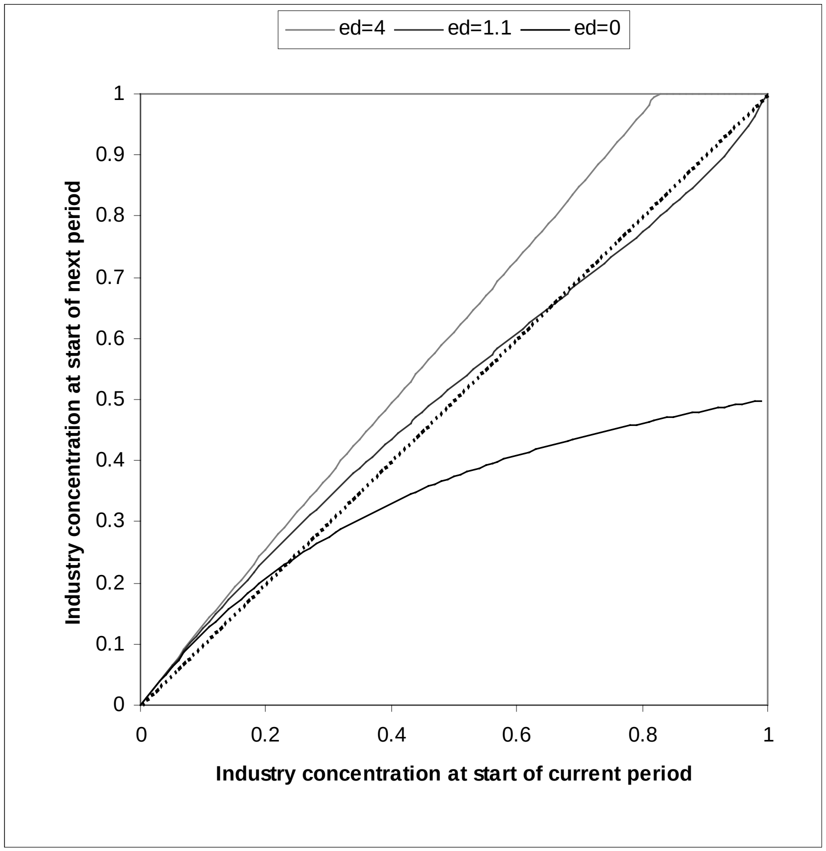


Figure 5: Industry Transition F(m°)
As β Varies; Elasticities ε_S=1, ε_D=0

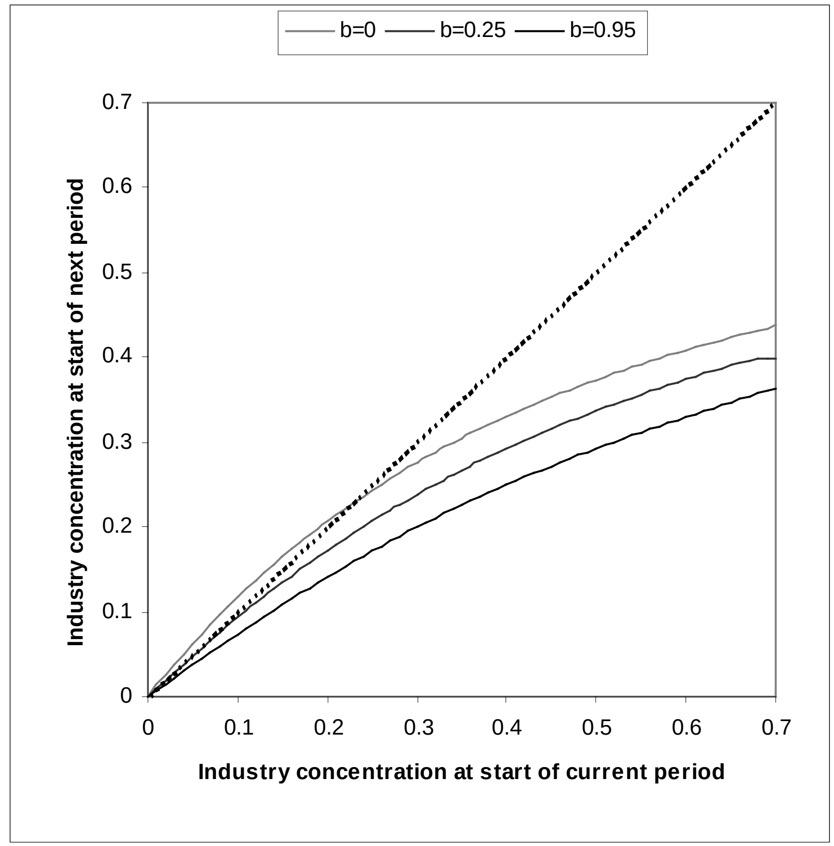


Table 1: Steady State Concentration m° With Initial State $m^\circ = 0.5$

Cost/demand elasticity	ε_D=0.5	ε_D=1	ε_D=1.2	ε_D=1.5	ε_D=2
ε_s=2	0.353	0.462	0.515	0.615	1
ε_S=1	0.377	0.586	0.722	1	1
ε_S=2/3	0.396	0.692	1	1	1
ε_S=1/2	0.410	0.778	1	1	1
ε_S=2/5	0.421	0.845	1	1	1

Table 2: Critical Value $\hat{\beta}(\varepsilon_D, \varepsilon_S)$
(Perfect competition is stable if and only if $\beta > \hat{\beta}(\varepsilon_D, \varepsilon_S)$.)

Cost/demand elasticity	ε_D=0	ε_D=.1	ε_D=1	ε_D =10
ε_S=.1	.25	.42	.76	.94
ε_S=1	.25	.27	.40	.68
ε_S=10	.25	.25	.27	.38
$\varepsilon_S=\infty$	.25	.25	.25	.25

Table 3: Critical Value $\overline{\beta}(\varepsilon_D, \varepsilon_S)$
(Monopoly is unstable if and only if $\beta > \overline{\beta}(\varepsilon_D, \varepsilon_S)$.)

Cost/demand elasticity	ε_D=2	ε_D=5	ε_D=10
ε_S=.1	.85	.92	.95
ε_S=1	.29	.55	.68
ε_S=10	.0004	.043	.16