Adjustable Rate Mortgages: Increasing Efficiency More Than Housing Activity

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Adjustable rate mortgages (ARMs)—mortgages with monthly payments that fluctuate with changes in interest rates—began to be widely issued after 1982. About this same time, housing activity rebounded at an unexpectedly healthy clip despite historically high nominal interest rates. Noting the coincidence of the growth of ARMs and the unexpectedly strong housing activity, many observers concluded that the widespread use of ARMs was primarily responsible for the strong 1982–84 housing activity. In this paper we argue, however, that there was little or no causal relationship between growing ARM issuance and the strong housing activity.[1] After providing some background, we question the argument made by those observers who attach critical importance to ARMs for stimulating the strong housing activity. We then suggest that ARMs became popular because they allow increased market efficiency; that is, ARMs permit borrowers and lenders to better share risks associated with fluctuating incomes and interest rates. Finally, using a statistically based forecasting model of housing activity, we provide some empirical evidence indicating that the housing spurt would have been about as strong without ARMs. The model finds that although nominal fixed rate mortgage (FRM) interest rates were high, effects associated with their decline during this period adequately account for most of the 1982–84 pickup in housing activity.

Background

Having been legalized nationally for federally chartered savings and loan institutions in 1981, ARMs began to be widely used to finance home purchases after 1982. By the end of 1982, an estimated $65 billion of ARMs were outstanding at major financial institutions. In 1983 alone, an estimated additional $39 billion of ARMs were added to this stock, accounting for one-third of the net growth of outstanding mortgage debt that year (Nothaft 1984, p. 448). In 1984, ARMs continued to be used heavily, accounting for around 60 percent of all conventional (non-FHA and -VA) home mortgage loans closed.

After 1982, housing activity also grew at a healthy clip, even though nominal interest rates were high compared with their historical average. Single-family housing starts, for example, grew from a seasonally adjusted annual rate of over 500,000 units in January 1982 to around 1 million in September 1984, reaching a high of about 1.3 million in February 1984. This growth occurred at a time when effective interest rates on conventional FRMs for newly built homes fell from a high of nearly 16 percent in May 1982 but never dropped below 12¼ percent.[2]

The rapid rate at which housing grew caught many analysts by surprise. For example, Goodman (1985, p. 1, n. 2) notes that "the average housing starts forecast for 1983 of 18 organizations polled by U.S. Gypsum Company at the beginning of 1983 was 1.44 million units.

[1] Similar conclusions have been reached by Esaki and Wachtenheim (1984–85) and Palash and Stoddard (forthcoming), although their methods differ from ours.

[2] The May figure includes some ARMs because a separate data series wasn't kept for them until July 1982.

Actual starts that year were 1.70 million." Similarly, *Blue Chip Economic Indicators* reported consensus forecasts for housing starts almost as low. And a single-family housing forecast we constructed for this study would have underpredicted seasonally adjusted housing starts during both 1983 and 1984.

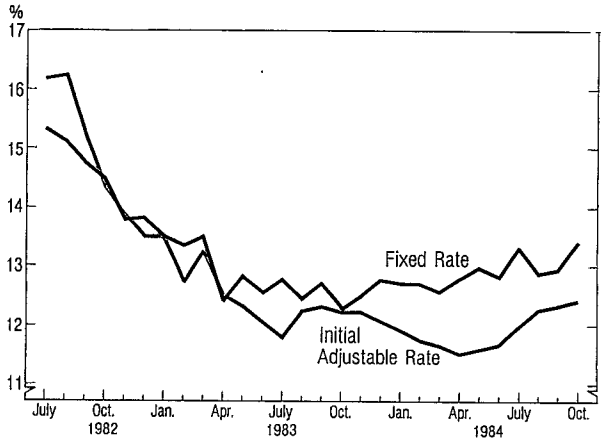
Noting that the growth of ARM use and the unexpectedly strong housing recovery concurred despite historically high FRM rates, some observers concluded that the advent of ARMs was responsible for the strong housing recovery. Trade publications printed stories asserting that ARMs were a major factor in increasing housing activity. One typical story quoted Paul W. Prior, chairman of the U.S. League of Savings Institutions: "There would be no housing activity to speak of without ARMs. . . . If home lending institutions could not make adjustable rate mortgages, we'd be forced to abandon housing just to survive in a deregulated savings world" (ARMs are the muscle behind housing, 1984, p. 45).

An Argument About ARMs . . .

Observers attributing critical importance to ARMs' role in the housing upswing often make the following argument. They assume that because the initial effective interest rates on ARMs (rates including fees and charges) were lower than those on FRMs, prospective home buyers might have wanted to buy higher-priced homes and/or a greater number of homes than they would have if ARMs weren't permitted (see Chart 1). These observers also believe that lenders were willing to accommodate buyers' plans to purchase more homes and/or higher-priced homes. The observers argue that to accommodate the purported higher housing demand, lenders would have applied similar loan qualification standards to ARM borrowers as they did to FRM borrowers, so that borrowers would indeed qualify for higher-priced homes if financed by ARMs rather than FRMs. As evidence that such accommodation occurred, observers cite the growing share that ARMs captured of conventional mortgage lending (see Chart 2).

Chart 1

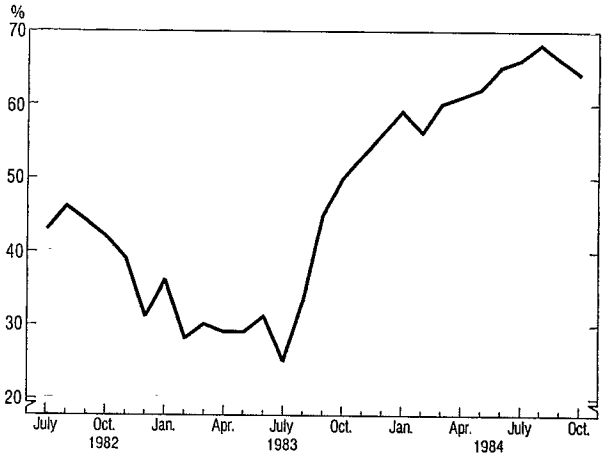
Mortgage Interest Rates for ARMs and FRMs*
(July 1982–October 1984)



*Average initial effective interest rates on conventional mortgages
Source: FHLBB, various dates

Chart 2

ARMs' Share of Conventional Mortgages Issued
(July 1982–October 1984)



Source: FHLBB, various dates

. . . And Some Reasons to Doubt It

At least two reasons exist for doubting the argument observers use to attribute such critical importance to ARMs' role in the 1982–84 increase in housing activity: The first reason questions the assumption that borrowers took much advantage of ARMs' increased borrowing power. The second stresses that factors other than how borrowers perceive the costs of ARMs should be considered before attributing critical importance to ARMs.

Borrowers Didn't Take Full Advantage of ARMs' Borrowing Power

First, it is not fair to assume that borrowers as a group took full advantage of the increased borrowing power

attributed to ARMs' lower initial effective interest rates (a point also noted in Goodman 1985, pp. 5–6). If borrowers had taken much advantage of the higher borrowing power inherent in the observers' argument that similar lending criteria apply to ARMs and FRMs, we would expect buyers' housing expense to income ratios to have remained roughly constant during this period. Instead, these ratios generally fell. The 1983 Home Buyer Survey shows that buyers generally spent less on housing as a fraction of their incomes in 1983 than in 1981 and 1979 (U.S. League of Savings Institutions 1984).[3] In particular, the median housing expense to income ratio for 1983 buyers was 23.1 percent, compared to 24.1 percent for 1981 buyers and to 24.3 percent for 1979 buyers. Moreover, the percentage of buyers spending only 20 percent or less of their incomes on housing rose from 28.5 percent in 1979 to 30.6 percent in 1981, then rose again sharply to 37.3 percent in 1983. With willing lenders, these buyers presumably could have qualified for larger mortgages if they had wanted, but evidently they did not. Data from the Panel Study on Income Dynamics also support the conclusion that home buyers typically don't take the biggest mortgage that lenders would allow them (Boehm and McKenzie 1983, p. 290, Table 1).

So why didn't borrowers take much advantage of the possible increased borrowing power of ARMs? They may not have due to the additional risk incurred by the possibility of interest-rate-induced payment fluctuations. One obvious source of risk is the possibility that the present value of mortgage payments (that is, the value of future payments in today's terms) would be higher with an ARM than with an FRM. Furthermore, we have theoretically shown that borrowers' evaluations of ARMs relative to FRMs depend not only on the present value of their expected payment streams but also on another source of risk: the correlation of borrowers' incomes with interest-rate movements and resulting mortgage payments (see Stutzer and Roberds 1985, Subsection 1B). According to our theory, even if some borrowers expect the present value of ARM payments to equal that of FRM payments, they will still value the FRM more than the ARM if they expect their incomes to be low when the ARM interest rate (and related mortgage payment) is high—that is, if they think their incomes and ARM interest rates will be negatively correlated. And this result favoring FRMs over ARMs (with expected payments of equivalent present value) holds even though the model assumes FRMs can't be refinanced with a new FRM when current rates fall below the previously contracted rate. This assumption forces FRM borrowers to risk not being able to take advantage of lower future interest rates.

Other Factors Were Involved: Improved Efficiency

A second reason to doubt the importance of ARMs in the housing upswing is that factors other than how borrowers perceive the costs of ARMs can affect the amount of housing demanded. For example, the disposable incomes of buyers and lenders, as well as lenders' aversion to interest-rate risk, eventually affect the amount of housing demanded.[4] Using a simple economic theory of incomplete markets, we have demonstrated the possibility that these other factors could combine in such a way as to nullify any effect that the authorization of ARMs would have on the amount of housing demanded (see Stutzer and Roberds 1985, Subsection 1C). According to this theory, rather than to increase housing demand, ARMs are issued and held to help lenders and borrowers hedge against future fluctuations in incomes and interest rates. Because the theory assumes that borrowers and lenders who voluntarily agree to ARMs are well-informed about the risks entailed, the authorization of ARMs potentially improves the efficiency by which risks associated with income and interest-rate fluctuations are allocated. This potential for improved efficiency (that is, the potential for both borrowers and lenders to be made better off) creates the major need for and effect of ARM use.

How can ARMs improve the efficiency of risk allocation between borrowers and lenders? When interest rates and mortgage payments decline, ARMs benefit borrowers because ARMs pass these declines along without forcing borrowers to incur the cost of refinancing an FRM. And when interest rates and payments increase, borrowers whose incomes also increase (that is, whose incomes are positively correlated with interest rates) will

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[3]These data were gathered from 371 member institutions of the U.S. League of Savings Institutions. Other lenders may have had different experiences.

[4]Another possible reason for doubting that ARMs are a major factor in the growth of housing activity is that lenders may view ARMs as riskier than FRMs and therefore adopt lending and pricing policies which partially nullify the impact of lower initial ARM rates. Lenders may believe that ARMs will generate less income in periods of declining interest rates than would FRMs with typical refinancing options. Or lenders may believe that ARMs carry higher default risks than FRMs. Either belief or both may lead lenders to make pricing and lending decisions in a way that dampens the impact ARMs have on housing activity.

However, while the view that lenders perceive ARMs as riskier than FRMs may have some credence, we have seen little evidence that lenders exercised this restraint in 1982 or through most of 1983. Later in 1984, though, major mortgage insurers sharply raised premiums on insured ARMs, forcing lenders' ARM pricing upward. Also, the Federal Home Loan Mortgage Corporation (Freddie Mac) tightened its standards for purchasing ARMs from original lenders, thus forcing some lenders to tighten their lending criteria. In August 1985, the Federal National Mortgage Association (Fannie Mae) also tightened its standards for purchasing ARMs from original lenders. So even if risk-induced lender restraint wasn't a factor in limiting ARMs' impact on housing activity before 1984, it probably has become a factor since then.

be hurt less when using ARMs. It is thus conceivable that many ARM borrowers will be made better off over the life of their mortgages as interest rates and payments fluctuate: the benefits accrued when interest rates and mortgage payments are lower may outweigh the costs incurred when rates and payments are higher. Lenders, in turn, are more likely to be made better off by issuing ARMs when their sources of loanable funds are predominantly short-term funds sensitive to interest-rate movements, as are most deposits. ARMs help these lenders reduce the risk of interest-rate fluctuations in two related ways. First, ARMs help them better match interest-rate-induced fluctuations in their costs of funds with the revenues earned from their mortgage holdings, thus reducing their profit risk. Second, ARMs help lenders reduce the risk that higher interest rates will erode the market resale value of their mortgage holdings, as happened with FRMs.

The prospects for improved efficiency make ARMs socially desirable, regardless of whether or not they substantially increase housing activity. That ARMs were intended to play a role in improving efficiency is suggested by the Federal Home Loan Bank Board's April 1981 authorization of ARMs in order to "allow the lender and the borrower the flexibility to agree upon the terms that will best suit their individual needs" (Amendment adopted . . . , 1981, p. 54). Our results help confirm that this purpose has been fulfilled.

In summary, we have suggested some reasons to doubt the argument that the ascension of ARM financing, with initial interest rates lower than FRM financing, was a major cause of the concurrent rapid growth of housing activity since 1982. Instead, the motivation behind and end result of the widespread issuance of ARMs may have been to improve the efficiency of risk allocation between borrowers and lenders.

If Not ARMs, Then What?

Because we doubt that ARM financing was a major factor in the 1982–84 growth of housing activity, an obvious question arises: What were major factors behind this growth? To help answer this question we constructed a simple statistical forecasting model of housing activity. After describing the model briefly (see Appendix A for details), we use it to support our hypothesis that ARMs were not a major factor in the housing upswing and to suggest that a major reason for the growth of housing activity in 1982–84 was effects associated with the general decline of FRM rates over 1982 and 1983 (shown in Chart 1). Specifically, we use the forecasting model to demonstrate that advance knowledge of actual FRM rates between January 1982 and October 1984 would have allowed us to forecast most of the actual growth in single-family housing starts and prices during this period—without knowing anything at all about ARM use. This suggests that if ARMs had not been permitted, more FRM lending and/or creative financing would have occurred to enable most of the growth in housing activity.[5]

Our Model of Housing Activity

Our forecasting model of housing activity uses the Bayesian vector autoregression (BVAR) methodology (described in Litterman 1985). Our BVAR forecasts initially included four monthly data series for the period January 1964–October 1984—the earliest and latest dates for which data were available. The series chosen were

- *Privately owned single-family housing starts,* taken as a measure of single-family housing market activity.
- *The real median price of new single-family houses,* used as an indicator of the inflation-adjusted average price of new homes sold; published in nominal form, the series was deflated using the consumer price index.
- *The average effective interest rate for conventional mortgages on new houses,* used as an index of mortgage interest rates. We also made use of the separate series, first available in July 1982, for the fixed and adjustable rate components of this average. We assumed that before July 1982, the average rate series and fixed rate series coincide.
- *The percentage change (difference in natural logarithms) in the consumer price index (CPI),* used as a measure of inflation.

We chose these four series not just for their power in forecasting housing activity but also to help us test whether or not ARMs were a significant factor in the rapid growth. (Further details about the data series are given in Appendix A.)

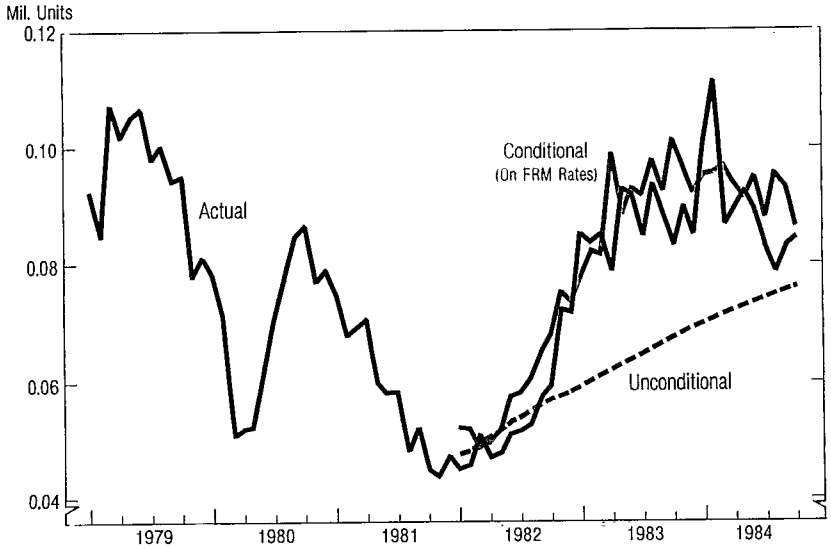
To help test for the possible effect of ARM financing on the level of housing starts, we first made two sets of forecasts for single-family housing starts and prices, both covering the period January 1982–October 1984 (see Charts 3 and 4). The first set includes forecasts that could have been made on December 1981; these forecasts only use data from December 1981 or earlier, before the large increase in ARM use. The forecasts from this set are

[5]By *creative financing* we mean a number of techniques borrowers and sellers adopt, in conjunction with FRMs, to help finance home sales. Such techniques include the use of contracts for deed and builder buydowns of FRM rates.

Charts 3 and 4

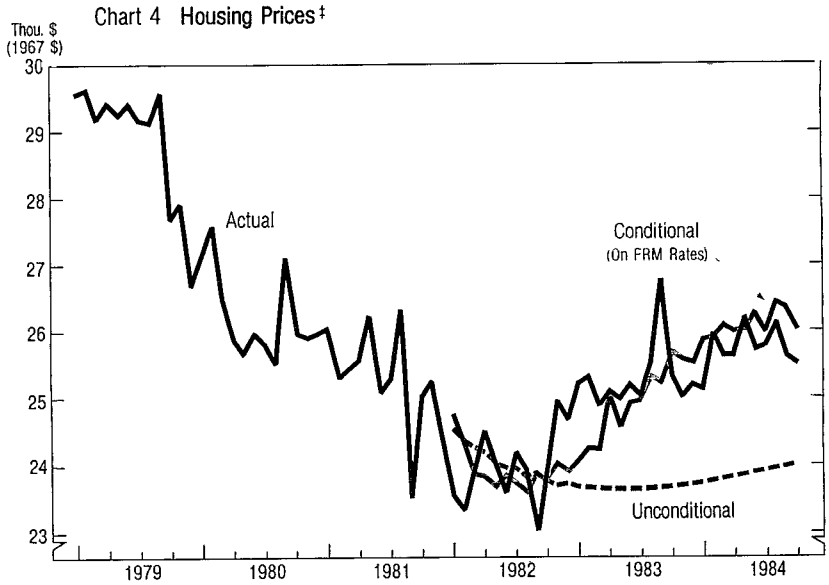
Our Model's Forecasts of Housing Activity*

Chart 3 Housing Starts†



† New privately owned single-family housing starts, seasonally adjusted monthly rate

Chart 4 Housing Prices‡



‡ Median sale price of new privately owned single-family houses, deflated by the consumer price index (1967 = 100)

*The unconditional forecasts use pre-ARMs data from January 1964 to December 1981; the conditional forecasts use the same data plus actual data for FRM rates from January 1982 to October 1984.

Sources of basic data: FHLBB, various dates; U.S. Department of Commerce

termed *unconditional forecasts.* The second set of forecasts uses all the data available for the first set plus the actual data for FRM rates between January 1982 and October 1984, when ARM use and housing activity grew rapidly. These forecasts are termed *conditional forecasts* because they are *conditioned* on future information—in this case, post-1981 FRM rates. These conditional forecasts show how much better the unconditional forecasts could have been if the future (January 1982–October 1984) FRM rates had been known.

Our Two Comparisons

We then used the unconditional and conditional forecasts to make two comparisons. The first compares the conditional to the unconditional forecasts to give an indication of the impact that falling FRM rates had on the post-1981 performance of housing starts and prices. The second compares the conditional forecasts to the actual data on housing starts and prices; this provides information about the effects that additional factors, including ARMs, may have had on housing starts and prices.

Our two comparisons are perhaps best explained by using a simple equation. Let A denote what we are trying to forecast—for instance, housing starts in February 1983. Let U denote the unconditional forecast and C denote the conditional forecast of A. Then, the error of the unconditional forecast, $(A - U)$, can be written as

$$(A - U) = (C - U) + (A - C).$$

That is, the error of the unconditional forecast is always equal to the difference between the conditional and unconditional forecasts, $(C - U)$, plus the error of the conditional forecast, $(A - C)$. The difference $(C - U)$ represents the impact of the additional information used to make the conditional forecasts. In our first comparison, this difference is attributed to the impact of unanticipated changes in FRM rates. In our second comparison, we examine the difference $(A - C)$. Due to the influence of additional factors other than FRMs, such as demographic factors, we would not expect $(A - C)$ to be zero even if ARMs had never existed. However, if the conditional forecast errors were unusually large or persistent, this would suggest that ARMs, perhaps in conjunction with these other non-FRM factors, had a significant impact on housing activity over the forecast period.[6] If the conditional forecast errors were fairly small and infrequent, this would suggest that neither ARMs nor these additional factors were major contributors to the housing upswing.

Our Results

The results of our first comparison, made by comparing the year-end 1981 conditional forecast (conditioned on actual post-1981 FRM rates) with the unconditional forecast, can be seen in Charts 3 and 4. This comparison shows how much higher and more accurately the model would have forecasted both single-family housing starts and real median home prices if the actual, post-1981 declining path of FRM rates had been known then. Since the conditional forecast is far more accurate than the unconditional one, it is evident that much information useful for accurately forecasting housing starts and prices is available from the actual fall in FRM rates—a fall which our model didn't predict.

The results of our second comparison, between the conditional forecasts and actual data shown in Charts 3 and 4, are that the conditional forecast doesn't systematically over- or underpredict either housing starts or prices during the 1982–1984 period. Because we do not find particularly large or persistent errors in the conditional forecasts, we can tentatively conclude that neither ARMs nor other non-FRM factors were major contributors to the unexpectedly strong housing recovery.

Of course, including other non-FRM factors in the model may help track housing starts and prices better. But since the conditional forecast error $(A - C)$ is already relatively modest, the slight increase in forecast accuracy earned by including other factors probably wouldn't justify attributing major significance to them.

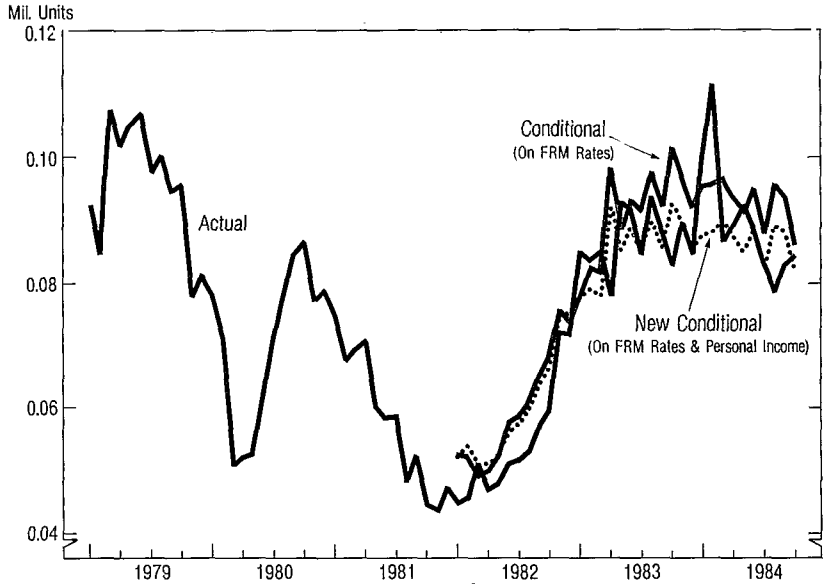
Another Test

Even so, we decided to test if the inclusion of another factor might substantially better the model's prediction of housing activity. We tried exchanging a U.S. personal income series for the model's inflation rate (CPI) series (which we dropped to conserve degrees of freedom in estimation), reasoning that anticipated growth in personal income, after the nation's recession bottomed out in November 1982, might have been a factor in the subsequent strong housing activity. However, the result of our test (shown in Chart 5) indicates that the new conditional forecast for single-family housing starts (conditioned on both FRM rates and personal income) is slightly lower and only somewhat more accurate than the old conditional forecast (conditioned only on FRM rates). We thus conclude that possibly higher-than-expected income growth probably wasn't a major factor in the unexpectedly strong housing activity: knowledge of

[6] It is possible that the presence of ARMs may have caused FRM rates to be lower than they otherwise would have been. If so, ARMs may also have (indirectly) influenced our first comparison—the difference $(C - U)$. But given the short length of the separate monthly interest-rate series (post-July 1982), we can't reliably test the hypothesis that ARMs' presence lowered FRM rates.

Chart 5

Including a Personal Income Series
in the Model's Conditional Forecast of Housing Starts*



*The housing start series and conditional forecast are the same as shown on Chart 3. The new conditional forecast is conditioned on FRM rates and U.S. personal income from January 1982 to October 1984.

Sources of basic data: FHLBB, various dates; U.S. Department of Commerce

actual income growth helped better predict housing growth, but not substantially so.[7]

Summary

Although the use of ARMs grew concurrently with housing activity between 1982 and 1984, we doubt that the former growth caused the latter. Rather than increasing housing activity, the growth of ARMs can primarily be explained by their role in helping borrowers and lenders better share risks associated with future fluctuations of interest rates and incomes. As such, ARMs are socially desirable regardless of whether or not they help spur housing activity. This is consistent with the Federal Home Loan Bank Board's rationale for authorizing ARMs to "allow the lender and the borrower the flexibility to agree upon the terms that will best suit their individual needs" (Amendment adopted . . . , 1981, p. 54). We primarily attribute the 1982–84 growth of housing activity to effects associated with the decline of FRM rates between 1982 and 1983. Our findings suggest that if ARMs had not been available, more FRM lending or creative financing or both would have taken place to fuel the growth of housing activity.

[7]Of course, lower interest rates (including FRM rates) may have helped predict the increase in housing activity because lower interest rates help cause higher personal income growth. If so, we wouldn't expect the inclusion of the personal income series to have improved the housing forecast. If this were the case, though, it is still reasonable to attribute the housing growth to the causative factor (lower interest rates) rather than to its effect (higher personal income growth).

Appendix A
The Bayesian Vector Autoregression Technique

In this appendix we describe the Bayesian vector autoregression (BVAR) technique used to construct our unconditional and conditional forecasts of housing activity. We discuss the technique's advantages and disadvantages, describe the four time series included in the model, and evaluate the model's forecasting performance.

There are advantages and disadvantages associated with the BVAR technique. The principal advantage is that the technique typically yields relatively good forecasts given relatively small inputs of human and computer time. For the purposes of our study, the major disadvantage is that the BVAR technique is a purely statistical one. While projections from BVAR models may be reasonably accurate, assigning unambiguous economic interpretations to these projections is often an elusive task.

Despite this serious limitation, we feel that the BVAR technique represents the best available methodology for the present study. Widely used alternative forecasting techniques include Box-Jenkins (univariate autoregressive moving average time series models) and structural econometric models. Box-Jenkins models, while simple to construct and estimate, use data only on one time series; as such, these models are of no use in constructing conditional forecasts. Structural models, based on economic theory, are preferable to BVAR models in the sense that it is easier to give economic meaning to their projections. However, constructing a structural model that is both theoretically and empirically valid can be a difficult and time-consuming task. For the purposes of preliminary data analysis, the BVAR technique might be thought of as a practical compromise between the two alternative techniques.

Accordingly, we estimated a four-variable (or four-equation) BVAR model for the following monthly data series:

1. Private single-family housing starts, monthly rate.
2. Median sale price of new privately owned single-family houses, thousands of current dollars.
3. National average effective interest rate on conventional mortgages for purchase of new homes, percent per year. After June 1982, the fixed and adjustable rate components of this series are available separately.
4. Consumer price index (CPI, 1967 = 100).

The source of series 1–3 was the Federal Home Loan Bank Board (FHLBB, various dates) and of series 4, the U.S. Department of Commerce. Series 1, 2, and 4 were seasonally adjusted using a procedure described in Amirizadeh 1985. Efficient seasonal adjustment of a quantity x_t (say, housing starts in January 1982) generally involves knowledge of future values of x_t (that is, housing starts in February 1982, March 1982, and so forth). For this reason, the forecasts reported in our study tend to be more accurate than those actually made over the forecast period (January 1982–October 1984). Each equation included 15 lags of all variables and a constant term. All series except the inflation series (CPI) were first converted

Theil U Statistics for the BVAR Model Forecasts*
(January 1976–December 1981)

Forecast Interval (months)	(1) Single-Family Housing Starts	(2) Real Median Price New Single-Family Homes	(3) Fixed Mortgage Rates	(4) Consumer Price Index	Number of Observations
1	1.014	.9303	.9726	.8950	61
2	1.017	.9351	.9882	.8980	60
3	1.031	.9617	1.0307	.8486	59
4	1.030	.9658	1.0090	.7866	58
5	1.018	.9896	.9634	.7735	57
6	1.008	.9880	.8749	.7479	56
7	1.001	.9955	.7738	.7757	55
8	1.002	1.0007	.7124	.7776	54
9	1.002	.9603	.6256	.8204	53
10	1.009	.9398	.6095	.8155	52
11	1.021	.9378	.6053	.8122	51
12	1.041	.9577	.6083	.7646	50

*Theil U statistics report the ratio of the root mean square error (RMSE) of the BVAR model's forecast to the RMSE of a naive forecast of no change in the (natural logarithms of the) series.

to natural logarithms. In the terminology of Doan and Litterman (1984), we placed a fairly "tight" prior over the model parameters. These priors were chosen because we felt that (the logarithms of) each of the four series could be approximated reasonably well as a random walk or as a random walk with drift.

The next step was to validate the model. To do so, we evaluated the out-of-sample (unconditional) forecasting performance of the model over the period January 1976 to December 1981. This was done for forecast intervals of 1 to 12 months. To avoid anticipating the effects of ARMs, no data from January 1982 or later were used in the evaluation.

Overall, the results of this evaluation indicate that our model performs at an acceptable level. Of particular interest are the Theil U statistics (see the table on page 17). These statistics report the ratio of the root mean square error (RMSE) of the model's forecast to the RMSE of a naive forecast of no change in the (natural logarithms of the) series. Except for housing starts, almost all of the Theil U statistics are below one; this indicates that our model outperforms the naive forecasting procedure. Unfortunately, in the case of housing starts, our model performs only about as accurately as the naive procedure, at least in terms of unconditional forecasts.

Experimentation suggests that our model's performance in predicting housing starts could be slightly improved by adding a short-term interest rate variable, such as a T-bill rate, as an exogenous variable in the VAR system. Our subsequent conditional forecasting experiments suggest, however, that this increase in forecast accuracy would largely be due to better unconditional forecasts of mortgage rates. (See Appendix B for details on how the conditional forecasts were made.) It is thus unlikely our conditional forecasts would be strongly affected by this complication. With the goal of keeping our study inexpensive, simple, and easily duplicable, we decided against constructing a more complicated model.

Ideally, our study also would have covered a greater number of indicators of housing sector activity. Increasing the number of series to be modeled by the BVAR technique, however, makes both forecasting and interpretation of BVAR models more difficult. Construction of a large BVAR model necessarily involves making some arbitrary modeling decisions inappropriate to the preliminary nature of our study. So we restricted our attention to two of the most widely followed aggregates: single-family housing starts and (real) median single-family home prices.

Appendix B
Unconditional and Conditional Forecasting With Vector Autoregression Models

This appendix provides a shortened version of a presentation of unconditional and conditional forecasting with vector autoregression (VAR) models in Doan, Litterman, and Sims 1984. The notation used is that of Sargent 1979, Chapter 10. (For an extensive discussion of forecasting with time series models, see Sargent 1979, Chapter 12.)

Consider an N-variate process x_t with the known VAR representation

$$(1) \qquad A(L)x_t = e_t$$

where x_t is an $N \times 1$ vector, $A(L)$ is an $N \times N$ polynomial matrix one-sided in nonnegative powers of L, and e_t is a vector of white noises.

Denote by Ω_t the set $\{x_t, x_{t-1}, \ldots\}$. The unconditional forecast of x_{t+k}, for $k > 0$, is defined to be the linear least squares projection of x_{t+k} on Ω_t, denoted

$$(2) \qquad P[x_{t+k} | \Omega_t]$$

or $\hat{x}_{t+k}$. It is well known that $\hat{x}_{t+k}$ can be derived recursively by the formula

$$(3) \qquad \hat{x}_{t+k} = -[A(L)/L]_+ \hat{x}_{t+k-1}$$

where $\hat{x}_{t+k} = x_{t+k}$, for $k \leq 0$, and where the notation $[\ ]_+$ means ignore negative powers of L.

Now let the vector x be defined by

$$(4) \qquad [x'_{t+1} \ldots x'_{t+K}]'$$

for $K > 1$. The unconditional forecast $P[x | \Omega_t] = \hat{x}$ can be obtained by successive applications of formula (3).

We now consider the conditional forecasting problem. Suppose that in addition to Ω_t, it is known that

$$(5) \qquad R'x = \tilde{r}$$

where R' is a known $J \times NK$ matrix of rank J, for $J \leq NK$, and $\tilde{r}$ is a known $J \times 1$ vector. The conditional forecasting problem is one of computing

$$(6) \qquad P[x | \Omega_t, R'x = \tilde{r}].$$

Since $\hat{x}$ is in the span of Ω_t, this is equivalent to calculating

$$(7) \qquad P[(x-\hat{x}) | \Omega_t, R'(x-\hat{x}) = \tilde{r} - R'\hat{x}].$$

Defining $v = x - \hat{x}$ and $r = \tilde{r} - R'\hat{x}$, the projection problem to be solved is that of calculating

$$(8) \qquad P[v | \Omega_t, R'v = r].$$

By the projection theorem (see Luenberger 1969, Chapter 3), the unconditional forecast error v is uncorrelated with Ω_t. Hence projection (8) reduces to

$$(9) \qquad P[v | R'v = r]$$

which we designate as u^*.

By definition, u^* solves the problem

$$(10) \qquad \min_u E(v-u)'(v-u)$$

subject to the constraint that $R'u = r$. Problem (10) is equivalent to the problem

$$(11) \qquad \min_u Ev'v + u'u$$

subject to the constraint that $R'u = r$, since u is nonstochastic and $Ev = 0$. Since $Ev'v$ does not depend on u, the last problem reduces to

$$(12) \qquad \min_u u'u$$

subject to the constraint that $R'u = r$. Problem (12) has solution

$$(13) \qquad u^* = R(R'R)^{-1}r.$$

The conditional forecast of x may then be recovered as

$$(14) \qquad P[x | \Omega_t, R'x = \tilde{r}] = \hat{x} + u^*.$$

The derivation provided is for a process x_t with mean zero. The formulas can be easily extended to processes with nonzero means.

To calculate our conditional forecasts, we used the RATS (regression analysis of time series) code given in Example 17.2 of Doan and Litterman 1984, which solves the programming problem (12). Having estimated $A(L)$ using data on $x_t, x_{t-1}, \ldots$ (with t = December 1981), we then proceeded as if $A(L)$ were known. Proceeding in this fashion ignores the impact of the additional information $R'x = r$ on the estimate of $A(L)$, but it saves significant amounts of computation time.

References

Amendment adopted—Bank board approves use of adjustable mortgage loan instrument. 1981. *Federal Home Loan Bank Board Journal* 14 (May): 54–56.

Amirizadeh, Hossain. 1985. Technical appendix for the Bayesian vector autoregression model of the U.S. economy. Available from the Research Department, Federal Reserve Bank of Minneapolis.

ARMs are the muscle behind housing. 1984. *Savings Institutions* (September): 45–51.

Boehm, Thomas P., and McKenzie, Joseph A. 1983. The affordability of alternative mortgage instruments: A household analysis. *Housing Finance Review* 2 (October): 287–94.

Doan, Thomas A., and Litterman, Robert B. 1984. Regression analysis of time series, Version 4.30 (RATS 4.30). Available from VAR Econometrics, P.O. Box 19334, Minneapolis, Minnesota 55419-0334.

Doan, Thomas; Litterman, Robert; and Sims, Christopher. 1984. Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews* 3 (1): 1–100.

Esaki, Howard, and Wachtenheim, Judy A. 1984–85. Explaining the recent level of single-family housing starts. *Federal Reserve Bank of New York Quarterly Review* 9 (Winter): 31–38.

Federal Home Loan Bank Board (FHLBB). Various dates. *News.* Washington, D.C.: FHLBB, Statistics and Analysis Division, Office of Policy and Economic Research.

Goodman, John L., Jr. 1985. Adjustable-rate home mortgages and the demand for mortgage credit. Working Paper Series, No. 41. Washington, D.C.: Board of Governors of the Federal Reserve System, Division of Research and Statistics, National Income Section.

Litterman, Robert B. 1985. A Bayesian procedure for forecasting with vector autoregressions and Forecasting with Bayesian vector autoregressions—Four years of experience. Research Department Working Paper 274. Federal Reserve Bank of Minneapolis.

Luenberger, David G. 1969. *Optimization by vector space methods.* New York: John Wiley.

Nothaft, Frank E. 1984. ARM debt outstanding by lender type and firm size. *Housing Finance Review* 3 (October): 447–55.

Palash, Carl J., and Stoddard, Robert B. Forthcoming. ARMS: Their financing rate and impact on housing. *Federal Reserve Bank of New York Quarterly Review.*

Sargent, Thomas J. 1979. *Macroeconomic theory.* New York: Academic Press.

Stutzer, Michael J., and Roberds, William. 1985. Variable rate loans increase efficiency but not necessarily borrowers' consumption of financed goods. Research Department Working Paper 285. Federal Reserve Bank of Minneapolis.

U.S. League of Savings Institutions, Economics Department. 1984. *Homeownership: Celebrating the American dream.* The 1983 Home Buyer Survey.

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