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Deficits, Interest Rates, and the Tax Distribution

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What is the relationship, if any, between higher government deficits and interest rates? Do permanently higher deficits necessarily imply that real interest rates will rise? Or can higher deficits be financed by more government borrowing without crowding out private capital, driving up interest rates, and producing high inflation? These questions have been addressed in three articles published previously in the *Quarterly Review* and reprinted in this issue. (See Sargent and Wallace 1981, Darby 1984, and Miller and Sargent 1984.) Missing from the earlier discussion, however, is adequate consideration of the role played by the distribution of taxes among economic agents. This paper attempts to highlight that role by showing that if the distribution of taxes is allowed to vary, it is feasible to finance a larger deficit by borrowing without affecting the interest rate. Moreover, it is not the level of the deficit per se but the distributional impact of its financing that may affect interest rates and hence the ability to finance the deficit by borrowing.

The Debate Thus Far

Before going further, let us briefly review the main points of the earlier articles. Sargent and Wallace (1981) consider an economy with a constant real growth rate γ where the government attempts to finance a given deficit (defined as the excess of government consumption g over taxes τ, both per unit of output) with money and interest-bearing bonds. They show that if the real interest rate on bonds r exceeds the growth rate ($r > \gamma$), then a policy of fixed money growth may be infeasible. For if the sum of direct taxes and the inflation tax on money falls short of government consumption, then the level of bonds per unit of output will grow without limit and will exceed the disposable income of savers. Consequently, the only choice for monetary policy is when to monetize the debt rather than whether. Using Samuelson's (1958) overlapping generations model, Sargent and Wallace construct some illustrative examples in which they show that a tight monetary policy maintained for some time and then followed by monetization of the debt may lead to uniformly higher inflation than a more accommodative policy held for the same length of time and followed by debt monetization.

In his response to Sargent and Wallace, Darby (1984) argues that if the real interest rate on bonds is less than the economy's real growth rate ($r < \gamma$), then the government earns positive seignorage on bonds and never needs to deviate from a policy of fixed money growth. (*Seignorage* is revenue the government earns from issuing money and bonds.) Since Darby finds that empirical evidence for the U.S. economy over the period 1926–81 favors his assumption, he concludes that the Sargent-Wallace argument is not empirically relevant.

In their reply to Darby, Miller and Sargent (1984) argue that historical evidence from a given regime of average deficits associated with the real interest rate being less than the real growth rate does not mean that the Sargent-Wallace argument is irrelevant. They point out that in an economy where the real interest rate depends on (among other things) the deficit, a change to a different regime involving higher deficits per unit of output may well push the real interest rate above the growth rate. When this happens, the monetarist arithmetic of Sargent and Wallace will indeed be unpleasant. Miller and Sargent argue that the rather large deficits projected for the U.S. economy over the near future may well be indicative of such a regime change.

Highlighting the Tax Distribution

Missing in the debate thus far is a discussion of the importance of the distribution of taxes.[1] To see its importance, we can relate this discussion to Wallace's (1981) analysis of open market operations. Wallace shows that open market operations will not affect either real or nominal variables under the following conditions:

a. The time path of government consumption is unchanged.

b. The time path of the deficit (defined as inclusive of interest payments and inflation-tax receipts) is unchanged.

c. The distribution of wealth among agents is unchanged.

In general, condition b implies different time paths for total direct taxes and hence different time paths for the deficit (defined as government consumption minus direct taxes). And yet, real or nominal variables need not be affected.[2]

This contrasts with Miller and Sargent's (1984) analysis in which such a result does not obtain. In their analysis, higher deficits will change the real interest rate and may well make it greater than the real growth rate even if initially (under a different deficit regime) the real interest rate is less than the growth rate. The key to this difference is condition c, the distribution of wealth among agents, which in turn depends on the distribution of the total tax burden. In the Miller-Sargent analysis, the distribution of taxes across generations is fixed; consequently, changes in deficits induce changes in lifetime wealth distributions and interest rates. However, if a change in the deficit is accompanied by a change in the distribution of taxes such that the wealth distribution is maintained, then interest rates need not be affected.

Assumptions and Propositions

In order to highlight the importance of the tax distribution in this debate, I need to use an economic model where the arguments of the earlier articles apply and where the tax distribution can be examined. For these reasons, I will work with a version of Samuelson's (1958) overlapping generations (OLG) model that is similar to the one used by Sargent and Wallace (1981).[3]

Using this model, I will demonstrate that under certain conditions the interest rate need not be affected by a change in the deficit. I first assume that government consumption exceeds total direct taxes. Then, without loss of generality, I assume that the growth rate is zero and construct an equilibrium in which the real interest rate is negative. (This is consistent with Darby's assumption, $r < \gamma$ with $\gamma = 0$). In the context of my model, I then prove the following propositions:

- A higher level of government consumption with unchanged total taxes can be financed by debt alone at an unchanged real interest rate.
- A lower level of total taxes with unchanged government consumption can be financed by debt alone at an unchanged real interest rate.

(For simplicity of argument, the propositions are solely concerned with debt finance; fiat money is not included in the model. The Appendix shows that fiat money can be included without affecting my conclusions.)

The Model

Here I describe the OLG model selected. I consider an economy with a constant population composed of agents who live for two periods. I assume that at each time $t(=1,2,\ldots)$ a single agent is born (the young at t) who is endowed with w units of a nonstorable commodity at t and who has no endowment in the second period.[4] The old agent at t, who was born at $t-1$ and is now in his or her second (final) period, has d_t units of government bonds, each of which represents a claim to one unit of time t consumption. I use the following notation:

g = government consumption (assumed to be constant over time)

$c_s(t)$ = consumption at t of the agent born at date s

$\tau_s(t)$ = lump-sum taxes at t on the agent born at date s

$\alpha \ln c_t(t) + (1-\alpha)\ln c_t(t+1)$ = the utility function of the agent born at t; $0 < \alpha < 1$

r_t = the real interest rate on government bonds from t to $t+1$.

The government budget constraint is

$$(1)\qquad g + d_t = \tau_{t-1}(t) + \tau_t(t) + [d_{t+1}/(1+r_t)].$$

This says that government consumption and the debt obligation to the old agent at time t must be met by taxes on the old agent at t, taxes on the young agent at t, and the proceeds of new bond sales to the young at t.

The young agent at t maximizes utility, subject to the following lifetime budget constraint:

$$(2)\qquad c_t(t) + [c_t(t+1)/(1+r_t)] = w - \tau_t(t) - [\tau_t(t+1)/(1+r_t)].$$

Given the log-linear specification of the utility function, the consumption demands for the young agent are given by

$$(3)\qquad c_t(t) = \alpha\{w - \tau(t) - [\tau_t(t+1)/(1+r_t)]\}$$

$$(4)\qquad c_t(t+1) = (1-\alpha)(1+r_t) \times \{w - \tau_t(t) - [\tau_t(t+1)/(1+r_t)]\}.$$

The old agent at time t cashes in bond holdings d_t, pays taxes $\tau_{t-1}(t)$, and consumes the rest. So the old agent's consumption demand is

$$(5)\qquad c_{t-1}(t) = d_t - \tau_{t-1}(t).$$

The consumption demands of the young agent (3)–(4), the old agent (5), and the government g, all at time t, must satisfy the economy's aggregate resource constraint at t:

$$(6)\qquad c_t(t) + c_{t-1}(t) + g = w.$$

By virtue of (1), (3), and (5), the above equilibrium condition can be rewritten as

$$(7)\qquad d_{t+1}/(1+r_t) = w - \tau_t(t) - c_t(t) = (1-\alpha)[w-\tau_t(t)] + [\alpha\tau_t(t+1)/(1+r_t)].$$

This says that the savings of the young agent at time t (the agent's endowment less current taxes and consumption) must equal the market value of debt sold by the government at t, since holdings of government debt are the only form of savings available for young agents.

Equations (1) and (7) describe the sequence of interest rates and government debt, given some assumptions about taxes and debt supplies. First, suppose that

$$(8)\qquad \tau_t(t) = \tau_1, \quad t = 1,2,\ldots$$

$$(9)\qquad \tau_{t-1}(t) = \tau_2, \quad t = 1,2,\ldots$$

and d_1 is taken as an initial condition. Then the solution is described by the following equations:

$$(10)\qquad 1 + r_t = \alpha\tau_2/[(g-\tau_1-\tau_2) + d_t - (1-\alpha)(w-\tau_1)]$$

$$(11)\qquad d_{t+1} = \alpha\tau_2[(g-\tau_1-\tau_2) + d_t] \div [(g-\tau_1-\tau_2) + d_t - (1-\alpha)(w-\tau_1)].$$

We now make two assumptions:

$$(12)\qquad 0 < \tau_2 < [(g-\tau_1)/(1-\alpha)] - (w-\tau_1)$$

$$(13)\qquad w > g > \tau_1 + \tau_2.$$

Under these assumptions there exists a positive, locally stable fixed point d to the difference equation (11).[5] This may be seen by putting $d_t = d_{t+1} = d$ in (11) and solving for d. This leads to the following quadratic equation in d:

$$(14)\qquad d^2 + d[g - \tau_1 - \tau_2 - (1-\alpha)(w-\tau_1) - \alpha\tau_2] - \alpha\tau_2(g-\tau_1-\tau_2) = 0.$$

By virtue of (12) and (13), it can be shown that equation (14) has one negative root, which is not economically relevant, and one positive root d, which can be shown to be locally stable.[6] From (10) and (11) we also have

$$(15)\qquad 1 + r_t = d_{t+1}/(g-\tau_1-\tau_2+d_t).$$

The steady-state interest rate r associated with the positive root d of (14) is therefore given by

$$(16)\qquad 1 + r = d/(g-\tau_1-\tau_2+d) < 1.$$

Hence, $r < 0$.

It can be seen from (14) that d depends not just on total taxes $(\tau_1 + \tau_2)$ but also on the distribution of total taxes across the young and the old agents. Consequently, from (16) it follows that the interest rate r also depends on this distribution. Thus, it is possible that an increase in government consumption g with no change in *total* taxes (so that the deficit is permanently higher) may be offset by a change in the distribution of taxes in such a way that the interest rate does not change.

Increasing Government Spending Without Changing Total Taxes or Interest Rates

I now use the model to demonstrate my proposition that government consumption can be increased ($g'>g$ as long as $g'<w$) with no changes in total taxes ($\tau_1'+\tau_2' = \tau_1+\tau_2$) by issuing more debt ($d' > d$) at an unchanged interest rate ($r' = r$).

Suppose that initially the economy is in a steady state with d and r given by (14) and (16). Let g', τ_1', and τ_2' be new levels of government consumption and taxes where

(17) $w > g' > g$

(18) $\tau_1' = \tau_1 + (g'-g)/r\{1 - [\alpha r/(1+r)]\}$

(19) $\tau_2' = \tau_2 - (g'-g)/r\{1 - [\alpha r/(1+r)]\}$.

Obviously, $\tau_1' + \tau_2' = \tau_1 + \tau_2$, so there is no change in total tax receipts. However, the distribution of the tax burden is different. The new policy calls for reducing taxes on the young and increasing taxes on the old (note that $r < 0$). It can be verified that this scheme leads to the same steady-state interest rate as before (namely, r) and to a higher level of government debt. Stability of the difference equation system for government debt and the interest rate is guaranteed if $\alpha > 0.5$ for any $g' > g$. (Of course, we must have $g' < w$.) This can be verified by checking that conditions (12) and (13) continue to hold for g', τ_1', and τ_2'. (Figures 1–3 illustrate the solution.)

The tax scheme (18) and (19) has the property that it distributes the burden of financing the higher level of government consumption equally between the young and the old, in the sense that the wealth distribution between young and old is unaffected. If we let (c_1,c_2) be the steady-state consumption allocations between the young and old, then in the original equilibrium

(20) $c_1 = \alpha\{w - \tau_1 - [\tau_2/(1+r)]\}$

(21) $c_2 = (1-\alpha)(1+r)\{w - \tau_1 - [\tau_2/(1+r)]\}$.

Equations (20) and (21) are simply the steady-state versions of (3) and (4). In the new equilibrium, since r is the same, we have

(22) $c_1' = \alpha\{w - \tau_1' - [\tau_2'/(1+r)]\}$

(23) $c_2' = (1-\alpha)(1+r)\{w - \tau_1' - [\tau_2'/(1+r)]\}$.

Noting that in a steady-state equilibrium the wealth of the old agent is simply equal to that agent's consumption, we get

(24)
$$\begin{aligned}&\{w - \tau_1 - [\tau_2/(1+r)]\}/c_2\\ &= \{w - \tau_1' - [\tau_2'/(1+r)]\}/c_2'\\ &= 1/[(1-\alpha)(1+r)].\end{aligned}$$

This shows that the ratio of the wealth of the young agent to that of the old agent is unchanged. In this sense, the wealth distribution is unaffected by the higher deficit, and the interest rate is unchanged. The relationship in (24) also shows that if the interest rate is different under two different deficit regimes, the wealth distribution must also be different. That is, it is not possible to affect interest rates without affecting the wealth distribution.[7]

Cutting Taxes Without Changing Government Spending or Interest Rates

I now demonstrate my proposition that, for a given level of government consumption g, it is possible to finance a higher deficit resulting from lower total taxes ($\tau_1''+\tau_2''<\tau_1+\tau_2$) by issuing more debt ($d'' > d$) at an unchanged real interest rate ($r'' = r$). As in the case with increased spending, the idea is to distribute the tax cut between the young and the old in a way that does not affect the wealth distribution and hence does not affect the interest rate.

Suppose that the economy is initially in a steady state with d and r given by (14) and (16). Consider the following alternative tax scheme that holds government consumption fixed:

(25) $\tau_1'' = \tau_1 + (\Delta\tau/r)$

(26) $\tau_2'' = \tau_2 - (1+r)(\Delta\tau/r)$

where $\Delta\tau > 0$. Obviously,

(27) $\tau_1'' + \tau_2'' = \tau_1 + \tau_2 - \Delta\tau < \tau_1 + \tau_2$.

This tax scheme keeps the wealth of a young agent unchanged at the previous interest rate, since

(28) $\tau_1'' + [\tau_2''/(1+r)] = \tau_1 + [\tau_2/(1+r)]$.

Hence, first-period consumption will be unaffected. But since first-period taxes are lower (because $r < 0$) by equation (25), savings will increase by $-\Delta\tau/r$. This increase in savings will accommodate exactly the additional debt that has to be issued to finance the tax cut, and the interest rate will be unaffected. This is simply the Ricardian doctrine (which says that the choice between tax and debt financing of government spending does not affect interest rates and consumption allocations) in an OLG framework where the tax cut is distributed among the agents in a way that does not affect wealth distributions. (Figure 4 illustrates the solution. In fact, a stronger conclusion can be demonstrated in the context of my OLG model, as shown in the accompanying box.)

Conclusion

I conclude that the proper argument for the monetarist arithmetic debate seems to be that it is not higher deficits per se that may alter the interest rate but, rather, how the burden of financing these higher deficits is distributed across heterogeneous agents. Equation (21) shows that the ratio of the wealth of the old agent, which is simply that agent's consumption c_2, to that of the young agent, $w - \tau_1 - [\tau_2/(1+r)]$, is related only to the interest rate. Consequently, as long as the distribution of wealth is unchanged across alternative equilibria, the interest rate cannot change.[8]

According to my model, the Miller-Sargent conclusions (that a shift to a different regime with permanently higher deficits will raise the interest rate and may make it exceed the growth rate) do not follow when the distribution of wealth is held constant. The model shows that a higher level of government spending can be financed by debt alone at an unchanged (and negative) interest rate and with unchanged total taxes, provided the distribution of the tax burden is adjusted to maintain wealth distributions. In the model, this requires reducing taxes on savers (the young) and increasing taxes on dissavers (the old), but leaving total taxes unchanged. Thus, any actual effect of higher government spending on interest rates may arise because distributional impacts are not being controlled for and not simply because the deficit is higher. Similarly, a cut in total taxes can be financed by debt alone at an unchanged (and negative) interest rate, provided taxes are cut (and raised) on individuals or groups in a manner that precludes distributional impacts. In the model, this requires cutting taxes on savers and raising taxes on dissavers to maintain the wealth distribution. Here again, any actual effect of tax cuts on interest rates may arise because distributional impacts are not being controlled for and not just because the deficit is higher.

In the Sargent and Wallace article as well as Miller and Sargent's, the authors implicitly assume that not only are total taxes fixed but that taxes on each individual or group are also fixed. Thus, alternative levels of the deficit correspond to alternative levels of government consumption. However, if an increase in the deficit is due to a cut in total taxes with unchanged government consumption, then presumably the tax cut is made in some fashion to all individuals each period. In either case, the wealth distribution will be affected and, along with it, the interest rate. When the deficit is higher, maintaining the wealth distribution requires a change in the distribution of taxes, in which case the interest rate need not change. The Sargent-Wallace and Miller-Sargent assumption about taxes may be more relevant for the recent across-the-board tax cuts than my own assumption of taxes being raised on one group while being lowered on another. Nevertheless, this should not detract from my theoretical point that it is not the level of the deficit per se but the distributional impact of its financing that may affect interest rates and hence the ability to finance the deficit by debt alone.

Thus, the level of the government deficit is a very inadequate measure of the impact of government budget policies on interest rates. Higher deficits can be associated with higher, lower, or unchanged real interest rates by suitably manipulating the wealth distribution through the tax system without affecting total taxes. As a result, we cannot, in general, draw a connection between aggregate measures of government activity and interest rates without considering the distribution of wealth. Thus, when the real interest rate is less than the real growth rate (or, in my analysis, when the real interest rate is negative, since I assume the growth rate is zero), higher deficits need not raise interest rates and impair the government's ability to use debt finance.

[1]In the ensuing discussion, keep in mind that *deficit* is defined as government consumption minus direct taxes; interest payments on the debt and the inflation tax on money are not counted. This definition is consistent with the usage in the previous articles.

[2]It should be stated that my analysis is different from Wallace 1981. The deficit policies examined here are not just asset exchanges. Government consumption may be different, which must lead to changes in private consumption, although it may or may not affect interest rates.

[3]I have selected an OLG model because this type of model can yield a real interest rate that may be above or below the real growth rate and can vary with different deficit policy regimes—so Darby's and Miller-Sargent's arguments would apply. Moreover, OLG models have heterogeneous agents, so taxes can be distributed differently among them. It also seems clear to me that the authors' previous discussion is carried out in the context of heterogeneous agent models. Sargent and Wallace 1981 contains examples of heterogeneous agent economies with overlapping generations in which the distribution of taxes across agents clearly matters. Finally, the model referred to in Miller and Sargent's reply (and described in Miller 1982) is also an OLG model.

For a number of reasons, I did not select another type of model commonly used to analyze the effects of deficit policies—that is, representative, infinitely lived agent models. In this type of model, all agents are identical, so the tax distribution can't be varied among different agents. Moreover, the real interest rate cannot be below the growth rate ($r \nless \gamma$), so neither Darby's argument nor part of Miller-Sargent's counterargument would apply. In addition, in this type of model (at least within the class where the representative agent has a constant rate of time preference), the interest rate is fixed, so Miller-Sargent's argument that higher deficit regimes result in higher real interest rates wouldn't apply.

These points about representative, infinitely lived agent models can be demonstrated in the following way: In such models, the economy consists of a representative, infinitely lived family which maximizes $\sum_{t=0}^{\infty}\beta^t U(C_t)$, where C_t is total consumption, $U(\cdot)$ is the utility derived in period t, β is a discount factor, and $0 < \beta < 1$. If r_t is the real interest rate from t to $t+1$, then a necessary condition for utility maximization is

$$MRS(C_t, C_{t+1}) = U'(C_t)/\beta U'(C_{t+1}) = 1 + r_t$$

where MRS is the marginal rate of substitution between C_t and C_{t+1}. As an example, suppose that $U(C) = \ln C$ and that a steady-state solution exists where $C_{t+1} = (1+\gamma)C_t$, so that the economy is growing at the rate γ. Then the steady-state interest rate r is given by

$$1 + r = C_{t+1}/\beta C_t = (1+\gamma)/\beta.$$

Since $0 < \beta < 1$, it must be that $r > \gamma$ and, further, that the steady-state value of r is independent of the government's budget policies.

[4]Sargent and Wallace (1981) allow for storage with constant returns to scale. The real return on bonds then cannot fall below the return on storage, though it may rise above the return on storage. I exclude storage so that the real interest rate is free to change with the policy regime.

[5]*Locally stable* means that if the initial level of debt d_1 is not too far from d, then the sequence of debts determined by equation (11) always converges to d, as t becomes large.

[6]Proofs of this assertion and others in this paper are contained in a Technical Appendix available on request to the Research Department, Federal Reserve Bank of Minneapolis.

[7]This result parallels the result in models with representative, infinitely lived agents in which increases in government spending simply crowd out consumption one-to-one but do not affect the steady-state interest rate or the capital stock. The difference here is that because of the OLG framework, the interest rate can be positive or negative.

[8]Obviously, it is also possible to have alternative equilibria with identical deficits and different wealth distributions and hence different interest rates. This can be obtained by simply changing the distribution of total taxes between the young and the old. Recall that from (14) and (16), the steady-state level of debt and the interest rate depend also on the distribution of taxes.

Appendix: Adding Money to the Overlapping Generations Model

To simplify my argument about the importance of the tax distribution for the discussion of deficits and interest rates, I omitted money from the overlapping generations (OLG) model used in my analysis. But it is not difficult to include money in the model, as is shown here. First, I show that when money is substituted for government debt, my two propositions still hold. Second, I show that when both money and bonds are included in the model, the propositions hold as well.

The Model With Money Only

Even though the OLG model does not contain money, it has the following (possibly surprising) implications for money finance of the deficit:

- An increase in government consumption with unchanged total taxes can be financed by money creation alone at an unchanged inflation rate.
- A cut in total taxes with unchanged government consumption can be financed by money creation alone at an unchanged inflation rate.

It is not difficult to understand these results if we remember that a positive level of government debt with a negative real interest rate is equivalent to a positive level of real money balances and a positive inflation rate. We simply assume that all government debt is in the form of fiat money, of which the initial old agent has M_0 units. We then identify government debt d_t with M_{t-1}/p_t and the interest rate $1 + r_t$ with p_t/p_{t+1}, where p_t is the price level at time t. The government budget constraint (1) assumes the form

$$\text{(A1)} \quad g = \tau_{t-1}(t) + \tau_t(t) + [(M_t - M_{t-1})/p_t].$$

Then, under the same assumptions as before, namely (12) and (13), there will exist a stationary monetary equilibrium with

$$\text{(A2)} \quad M_{t-1}/p_t = d$$

and

$$\text{(A3)} \quad p_t/p_{t+1} = 1 + r.$$

Given M_0, equation (A2) with $t = 1$ determines the initial price level. The inflation rate (which will be positive, since $r < 0$) is given by (A3); it determines the entire price sequence. The money supply path is determined by (A1) or (A2). The propositions about debt finance in my analysis can now be translated in terms of money finance.

The Model With Money and Bonds

Fiat money can also be easily included with government bonds in the OLG model without affecting my conclusions. I do this in a manner that parallels Sargent and Wallace 1981. I assume that in each period another agent is born who has y units of endowment in the first period and none in the second period. The old agents at date 1 hold M_0 units of money (in addition to government bonds), and the government pursues a policy of fixed money growth, denoted by θ:

$$\text{(A4)} \quad M_t = (1+\theta)M_{t-1}, \quad t = 1, 2, \ldots .$$

I then assume that $w >> y$, so that the second agent is much poorer than the first; that bonds are large-denomination obligations which the poor agent cannot afford (but the rich can); and that

$$\text{(A5)} \quad 1 + r > 1/(1+\theta).$$

That is, the real return on bonds is greater than the real return on money. Finally, I assume that intermediation between large-denomination bonds and small-denomination currency is prohibited and that the poor agent never faces any taxes. Under this scenario, the markets for money and bonds will be completely segmented—that is, the rich hold bonds and the poor hold money.

Assuming that the poor agent has the same preferences as the rich, the demand for real balances is given by

$$\text{(A6)} \quad \text{demand} = (1-\alpha)y = M_t/p_t = \text{supply}.$$

In combination with (A4), the time path of price levels is determined by

$$\text{(A7)} \quad p_t = (1+\theta)M_{t-1}/(1-\alpha)y, \quad t = 1, 2, \ldots .$$

The government budget constraint is modified to

$$\text{(A8)} \quad g + d_t = \tau_t(t) + \tau_{t-1}(t) + [d_{t+1}/(1+r_t)] + [(M_t - M_{t-1})/p_t].$$

Or, equivalently,

$$\text{(A9)} \quad \{g - [\theta(1-\alpha)y/(1+\theta)]\} + d_t = \tau_t(t) + \tau_{t-1}(t) + [d_{t+1}/(1+r_t)].$$

This is essentially the same as constraint (1), except what was previously referred to as government consumption should now be reinterpreted as the excess of government consumption over the inflation-tax receipts from money creation. However, since none of that analysis involved changing θ or taxes on the poor agent, the results hold.

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Figures 1–3

Increasing Government Spending With the Same Total Taxes $(\tau_1' + \tau_2' = \tau_1 + \tau_2)$ and Interest Rate

Figure 1 Higher government spending reduces feasible consumption allocations, given the economy's aggregate resource constraint $c_1 + c_2 + g = w$.

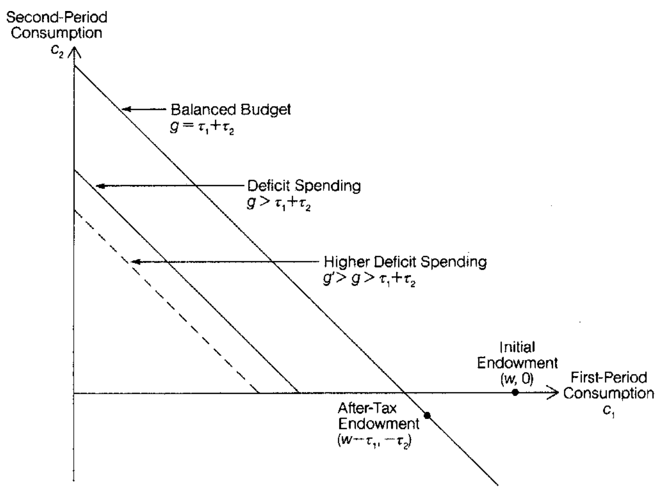


Figure 2 At the same interest rate, the new tax distribution reduces an agent's consumption, given the agent's budget set, $c_1 + [c_2/(1+r)] = w - \tau_1 - [\tau_2/(1+r)]$.

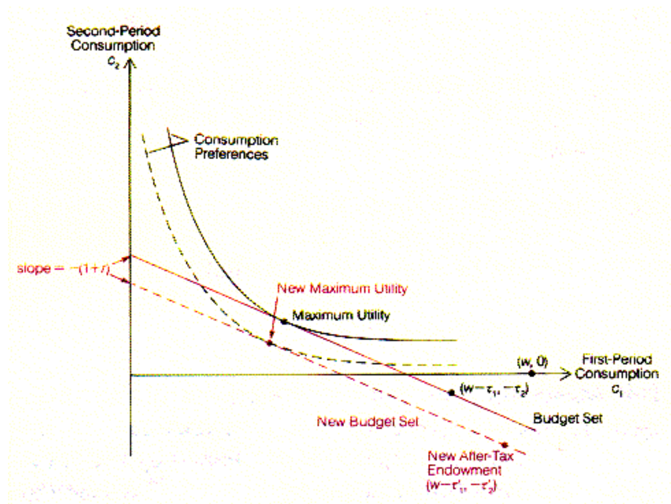


Figure 3 The equilibrium interest rate is unchanged with higher government spending and the new tax distribution.

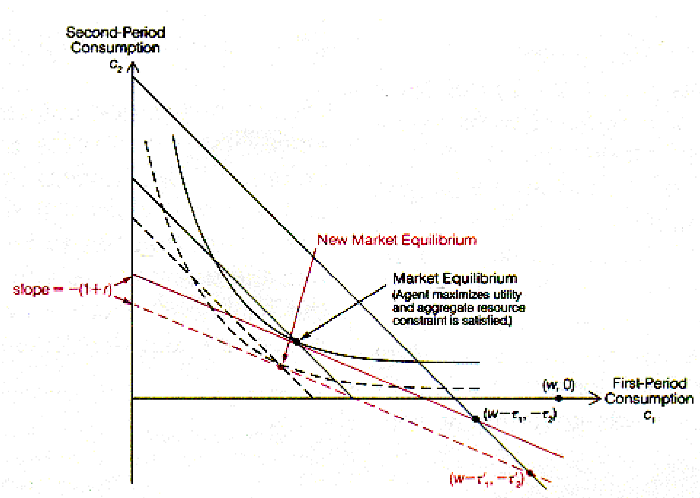
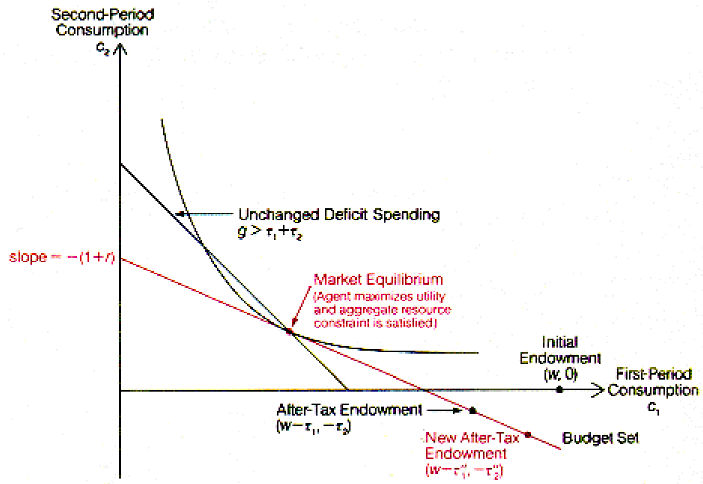


Figure 4

Cutting Total Taxes $(\tau_1'' + \tau_2'' < \tau_1 + \tau_2)$
Without Changing Government Spending and the Interest Rate



For Every Deficit, Taxes Can be Distributed Such That r < 0

My overlapping generations model can also be used to demonstrate that for each new deficit, there is a way to distribute taxes such that the new equilibrium interest rate remains less than zero. Suppose that a constant stream of government consumption g and total taxes τ are given such that $w > g > \tau$. Then there exists a distribution of total taxes among the young and the old [$\tau_t(t)$ and $\tau_{t-1}(t)$, respectively, such that $\tau_t(t) + \tau_{t-1}(t) = \tau$] which will support a competitive equilibrium with a negative real interest rate and a positive level of debt. (The construction of such an equilibrium is depicted in the accompanying figure.) Thus, in the face of a changing deficit, even if the old interest rate cannot be sustained as an equilibrium, it is always possible to sustain a new—and still negative—equilibrium interest rate without changing the new level of total taxes.