# *Some Pleasant Monetarist Arithmetic*

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Sargent and Wallace (1981) are widely regarded to have demonstrated that monetary policy cannot be manipulated independently (exogenously) when the growth path of government expenditures and the tax structure are both fixed. More succinctly, Sargent and Wallace maintain that the only choice available to the central bank is not whether to monetize a government deficit but when—now or later. This result can be viewed as a generalization of the Blinder and Solow (1973, 1974), Tobin and Buiter (1976), and Steindl (1974) analyses of the stationary state when it is assumed that the monetary base is increased while government spending and the tax rate are fixed, so that government borrowing is adjusted passively via open market operations.[1] Although Sargent and Wallace's argument appears persuasive to such authors as King and Plosser (1983), I believe it is seriously wrong as a guide to understanding monetary policy in the United States. To prove my point, this paper first demonstrates that whether or not the government can independently manipulate money, spending, and taxes is not a theoretical question. Then I present evidence that, at least in the United States, the government can indeed independently manipulate all three instruments, with government debt adjusting in a passive but stable manner.

## Pleasant Arithmetic Reverses a Key Sargent-Wallace Assumption

Miller (1983) has derived a version of the government budget constraint which is useful for studying the long-run growth equilibrium of the economy. Simplifying the notation of Miller's equation (6), we can rewrite this constraint as

$$(1) \qquad G - T = \mu M + (\delta - r)D$$

where the following are expressed as ratios to net national product (NNP),

$G$ = government expenditures (excluding interest payments and taxes thereon)

$T$ = government tax receipts

$M$ = the monetary base

$D$ = the stock of government debt

and where the following are rates per unit of time,

$\mu$ = the growth of the monetary base

$\delta$ = the growth of real government debt

$r$ = the real after-tax interest rate.[2]

The constraint states that the excess of spending over taxes must be financed either by base money creation or by borrowing in excess of the amount needed to pay the real

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[1] This paper does not attempt to comment on the relevance of the balanced-budget condition within the stationary state. See, however, Fischer 1976 and Auerbach and Rutner 1977 on this point.

[2] See Darby 1975. Further discussion of the use of the after-tax real yield appears later in this paper.

after-tax interest on the government debt.[3]

The question raised by Sargent and Wallace's article is whether only one value of money-creation revenue $\mu M$ exists for which debt will be a stable fraction of NNP.[4] This question can be formalized by asking whether a steady-state equilibrium exists and is stable for alternative values of $\mu M$. The steady-state equilibrium debt-to-NNP ratio $\bar{D}$ is found from equation (1) to be

$$(2) \qquad \bar{D} = (G - T - \mu M)/(\gamma - r)$$

where

$\gamma$ = the growth rate of real NNP

and

$\gamma = \delta$ if the debt-to-NNP ratio $D$ is constant.

Equation (2) says that if the government is spending more than it collects in explicit taxes and the inflation tax, there can still be a constant debt-to-NNP ratio if the real NNP growth rate $\gamma$ exceeds the real after-tax interest rate $r$. However, if $r$ exceeds $\gamma$, then any positive excess of $G$ over $T + \mu M$ would indeed cause $D$ to grow without limit. Sargent and Wallace simply assume that $r$ exceeds $\gamma$, and hence they inevitably conclude that the government cannot independently choose $\mu$, $G$, and $T$.

In contrast, my analysis proceeds here on the assumption that $\gamma$ is greater than $r$. (I shall argue later in this paper that empirical evidence supports this assumption.) My basic reasoning is that the government will borrow more than enough to make interest payments on its debt if the debt-to-NNP ratio $D$ is constant, and that this net borrowing $(\gamma - r)D$ increases with $D$.[5] As a result, higher deficits $G - T - \mu M$ will be associated with higher debt-to-NNP ratios, but these deficits can be financed indefinitely as a matter of arithmetic unless the real interest rate were equal to or greater than the growth rate of real income.

To check that the economy will in fact move toward the equilibrium debt-to-income ratio, suppose that the actual value of $D$ differed from its steady-state value $\bar{D}$. The growth rate of $D$ is $\delta - \gamma$, which is the difference in the growth rates of $D$'s numerator and denominator. Straightforward manipulations and the assumption of either perfect foresight or indexed government bonds imply the growth rate relation[6]

$$(3) \qquad \delta - \gamma = (\gamma - r)\,[(\bar{D} - D)/D].$$

That is, the growth rate of the debt-to-income ratio will be positive if the actual $D$ is less than its steady-state value $\bar{D}$, and negative if $D$ exceeds $\bar{D}$. So $D$ will gradually converge to $\bar{D}$ even if the economy were to start from another position, such as that which might result from cyclical deficits, wars, short-run monetary or fiscal policy, or changes in the underlying trend values of $G$, $T$, or $\mu$ which define the steady-state equilibrium.

### *An Example*

To illustrate that alternative monetary policies are consistent with a given fiscal policy, consider the following simple example where

$G = 0.22$

$T = 0.18$

$\mu = 0.10$/year

$M = 0.10$ year

$\gamma = 0.04$/year

$r = 0.02$/year.

By substituting these values into equation (2), we can determine the steady-state debt-to-income ratio as follows:

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[3]The standard national income accounting definition of the deficit counts as government borrowing and private saving that portion of after-tax nominal interest which represents an adjustment for decline in the real value of the nominal debt. In those terms, we would include in equation (1) the growth rate of the nominal debt $\delta + \pi$ and the nominal after-tax interest rate $r + \pi$, where the inflation rate $\pi$ cancels. See Jump 1980 and Darby and Lothian 1983. Miller's equation (6) substitutes the steady-state condition that the growth rates of real NNP and real debt are equal, but we leave the equation in this form to analyze behavior out of full steady-state equilibrium.

[4]The fraction of NNP which people desire to hold as money is a decreasing function of the nominal interest rate and hence $\mu$. In the relevant range, $\mu M$ increases with increases in $\mu$, but not proportionately so.

[5]An alternative term for net borrowing $(\gamma - r)D$ would be *negative debt service.*

[6]These manipulations begin with the identity

$$\delta - \gamma \equiv [(r+\pi)D + G - T - \mu M - (\gamma+\pi)D]/D$$

which was obtained by taking the time derivative of the natural logarithm of $D$ where the perfect foresight or indexing assumption allows us to express the nominal after-tax interest rate as the sum of the corresponding real interest rate and the actual, rather than expected, rate of inflation. (In the steady state, there is no need to distinguish actual from expected inflation.) Then, we have

$$\delta - \gamma = [(\gamma - r)/D] \times [(G - T - \mu M)/(\gamma - r)]$$

from which equation (3) follows by substitution of equation (2).

$$\bar{D} = [0.22 - 0.18 - (0.10/\text{year})(0.10 \text{ year})]$$
$$\div [(0.04/\text{year}) - (0.02/\text{year})]$$
$$= 0.03/(0.02/\text{year})$$
$$= 1.5 \text{ year.}$$

Suppose that the Fed decided to increase money growth to $\mu' = 0.20$/year and that this induced $M$ to fall to $M' = 0.09$ year. Then the new equilibrium debt-to-income ratio is

$$\bar{D}' = [0.22 - 0.18 - (0.20/\text{year})(0.09 \text{ year})]$$
$$\div [(0.04/\text{year}) - (0.02/\text{year})]$$
$$= 0.022/(0.02/\text{year})$$
$$= 1.1 \text{ year.}$$

When this policy is initiated, the growth rate of the debt-to-income ratio, using equation (3), would be

$$\delta - \gamma = [(0.04/\text{year}) - (0.02/\text{year})]$$
$$\times [(1.1 \text{ year} - 1.5 \text{ year})/1.5 \text{ year}]$$
$$= -0.0053/\text{year.}$$

That is, over the first year of the new policy, the debt-to-income ratio would fall by approximately $-0.0080$ year [by calculating (1year) $\times$ ($-0.0053$/year) $\times$ (1.5 year)] to the new level of 1.492 year. The rate of decline would decrease as $D$ asymptotically approached $\bar{D}' = 1.1$ year.[7]

Thus, the government budget constraint does not pose any problems for the existence or stability of the steady-state equilibrium as money growth is varied exogenously with fiscal policy fixed. Similarly, either government spending or tax rates can be varied exogenously when the other fiscal variable and monetary policy are held unchanged. In this way, the standard macroeconomic practice of varying fiscal or monetary instruments while allowing government borrowing to adjust passively is shown to be consistent with a stable steady-state equilibrium.

## Empirical Evidence Favors the Pleasant Arithmetic

Like Sargent and Wallace's, my discussion thus far has been basically an arithmetic exercise. In this section, however, I argue that the empirical evidence favors the relevance of my assumptions for the U.S. economy. I base my argument upon two substantial differences between the respective arithmetic exercises: (1) Sargent and Wallace use *before-tax* real yields instead of *after-tax* real yields, and (2) they assume that the relevant real yield exceeds the growth rate of real income.

### *Differences About the Relevant Real Yield*

The differences over which real yield should be compared to real income growth are partly semantic and partly substantive. The semantic difference depends on how an exogenous fiscal policy is defined. Sargent and Wallace define an exogenous fiscal policy as a fixed path for the difference between government spending and taxes (exclusive of money or debt creation and interest payments) measured in terms of real goods. I instead hold the levels of each of these variables (and hence their difference) constant as a fraction of real income. Thus, if decreased money growth reduces real income, it would also reduce the level of the future real deficit, based on my assumption of constant deficits as a fraction of income.

This difference in the way exogenous fiscal policy is defined is relevant only if lower money growth (and hence a higher debt-to-income ratio) reduces real output, as supposed by Sargent and Wallace. They argue that crowding out will occur because, in their life-cycle framework, more government debt means that less wealth will be held in the form of capital. Thus, tax receipts on capital returns go down as tax receipts on government debt go up.[8]

Suppose instead that individuals are fully rational and care about their children as themselves. In that kind of world, government accounts are consolidated into those of

[7]Note, however, that in the absence of perfect foresight or a prior refunding into indexed bonds of long-term bonds (see Darby and Lothian 1983), this adjustment will be much faster as the real value of the existing bonds and debt service drops.

[8]Tobin (1965) proposed a different mechanism by which inflation might reduce the private capital stock. In either case, as firms devote less inputs to conserving cash balances, improvements in the aggregate production function would tend to offset, eliminate, or dominate this capital stock effect, so that the effect of money growth on real output is theoretically ambiguous. I have assumed elsewhere (in Darby 1979a) that the production function effect dominates, so that lower inflation rates increase real output.

the individuals whom it represents, so that whether the government finances by taxes or bonds is irrelevant to individual choices about consumption and the accumulation of physical capital. Measured saving will equal the unaffected capital accumulation plus however many new government bonds are issued instead of tax receipts. But individuals will not be concerned about how many IOUs they are writing to themselves.[9] It should be noted that, given this latter view of saving behavior, the real interest rate is unaffected by the level of the debt-to-income ratio.[10]

Thus, there are good reasons to suppose that the difference in the ways we define exogenous fiscal policy is not a substantive one after all. The fall in private capital, which Sargent and Wallace associate with higher levels of the debt-to-income ratio, need not occur. Nonetheless, it will be shown below that even the before-tax real yield on government securities has been generally well below the growth rate of real income. In that case, even in the Sargent and Wallace world, exogenous variations in the deficit need not be monetized by the central bank.

### *Real Yields Versus Real Growth*

As anyone who has ever looked at before-tax real yields on government securities is aware, it is a simple matter to show that long-term before-tax real yields have not approached corresponding growth rates of real output. It follows directly that after-tax real yields must be even less. This is not to suggest that the real return to capital in the economy is less than the growth rate of real output; but the real rate of return on government bonds and bills is clearly far below this average social return. Presumably, the difference between government and private returns reflects both nonpecuniary services and a very low correlation with the market return, but that really is not at issue in understanding the implications of the government budget constraint.

Ibbotson and Sinquefield (1982) have compiled before-tax real rates of return for U.S. government bonds and Treasury bills from 1926 to 1981. The arithmetic means of the yields for long-term government bonds and Treasury bills are 0.3 and 0.1 (geometric means: −0.1 and 0.0) percent per annum, respectively.[11] So even if all holdings were tax exempt, the experience of the last 55 years suggests that the after-tax real yield on government securities has been nowhere near the 3.0 percent per annum average growth rate of real income over the same years.[12]

It would be possible to increase the estimated real yield somewhat, but I have been unable to find any study that indicates an average real yield on government securities as high as 3 percent, even without any allowance for income taxes. Taking account of income taxes would lower these estimates; so there seems to be no doubt empirically that for the United States, the growth rate of real income exceeds the after-tax real yield on government securities.

## A Possible Reconciliation

The point of this paper is a technical one: In the United States, dynamic inconsistencies do not result from treating government expenditures, taxes, and money growth as simultaneously exogenous. A current deficit is therefore not per se inflationary in the sense of requiring future increased money growth, as claimed by Sargent and Wallace.

This conclusion would not hold for all economies, nor need it always hold for the United States. Suppose, for example, that as the ratio of goverment debt to income—and hence to physical capital—rises, the yield on government debt rises toward that of physical capital instead of remaining constant, as assumed above and by Sargent and Wallace. Then, if the equilibrium debt-to-income ratio were to increase to the point that the after-tax real yield on government securities equalled or exceeded the growth rate of real income, the economy would cross over to the explosive character analyzed by Sargent and Wallace. While this may have occurred for other countries in the past, the United States does not yet seem near that point.

To see this, first consider the fiscal 1983 deficit, estimated at $208 billion by the U.S. Council of Economic Advisers (1983, p. 26). If we allow for a cyclical component based on moving from the assumed 10.7 to 6.0 percent unemployment rate, the structural deficit would be about $117.5 billion less—that is, about $90 billion. This amount is only $5 billion more than actual fiscal 1982

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9 White (1978), Darby (1979b), and Kotlikoff and Summers (1981) all report evidence that bequest assets dominate life-cycle assets in total U.S. wealth, and this finding supports the assumption of concern about the welfare of one's children. Barro (1974, 1978), Kochin (1974), and David and Scadding (1974) all present evidence in support of the ultrarational or Ricardian view. Note that if the government finances a tax cut with increased borrowing, saving increases not in anticipation of future increased taxes but in anticipation that, otherwise, total NNP would fall.

10 Plosser (1982), for example, finds that asset prices are unaffected by the extent to which a given level of government expenditures is financed by borrowing instead of taxes.

11 The corresponding nominal yields were 3.1 and 3.1 (arithmetic) and 3.0 and 3.0 (geometric).

12 Computed from real GNP data in Darby 1984 (Table A-20) and in FR Board 1983 (p. A52).

interest payments. So even without taking account of the large offsetting state government surpluses, there is no evidence of substantial differences between long-term government spending (exclusive of interest) and net taxes. Furthermore, current ratios of government debt to income are far below the 1946 value of 1.1.[13]

## Conclusion

Sargent and Wallace's propositions should not be generally applied in analyses of the U.S. economy or similar economies. Where the propositions are applied, they should be justified by evidence that the after-tax real yield on government bonds really does exceed the growth rate of real income or would do so under the circumstances being considered. It is hardly surprising that arithmetic alone cannot give a real answer to a substantive economic question.

[13]High ex post real interest rates experienced during 1981–82 appear to be a result of a slowing of inflation (compare 1929–33) and not a matter of a regime change to unprecedentedly high deficits.

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